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Newton Fractals[1]

Today we study convergence of the Newton-Raphson method for the cyclic polynomial of degree 5. I suggest you work in pairs, splitting the last four questions between you.

Q0. Find the five roots of the cyclic polynomial $z^5 - 1 = 0$. Put them into an array.

Q1. Divide the square $[-1.5; 1.5] \times [-1.5; 1.5]$ into 1500^2 equal squares with sidelengths 0.001. Apply the Newton-Raphson method to every vertex (500 iterations in the loop is sufficient). Associate a root (its number) to every vertex. The output should be a 1500×1500-matrix, where each element contains the root's number. You need to treat a point (a, b) as a complex number $a + i \cdot b$.

Q2. Plot in five different colours five domains corresponding to convergence to different roots (on the same plot). Use options "with dots" `'.'` and `'MarkerSize'` which should be set to 1.

Q3. Plot the boundary of the domains, that is all points that have a neighbour of a different colour. Highlight those where the absolute value of the derivative is greater than 1:

$$\left|\frac{\mathrm{d}}{\mathrm{d}z}\left(z - \frac{z^5 - 1}{5z^4}\right)\right| = \left|\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{4}{5}z + \frac{1}{5z^4}\right)\right| = \frac{4}{5} \cdot \left|1 - \frac{1}{z^5}\right| > 1.$$

Q4. The Heron formula for the area of a triange via sidelengths reads

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}.$$

Find a triangle inside the square $[-1.5; 1.5] \times [-1.5; 1.5]$ of maximal area with all three vertices of the same colour.

Q5. The line passing through the points (a_1, b_1) and (a_2, b_2) on the plane is given by the equation

$$y - x \cdot \frac{b_1 - b_2}{a_1 - a_2} + \frac{b_2 a_1 - b_1 a_2}{a_1 - a_2} = 0$$

It divides the plane into two parts and the points that belong to the same part satisfy one of the two strict inequalities

$$y - x \cdot \frac{b_1 - b_2}{a_1 - a_2} + \frac{b_2 a_1 - b_1 a_2}{a_1 - a_2} < 0 \quad \text{or} \quad y - x \cdot \frac{b_1 - b_2}{a_1 - a_2} + \frac{b_2 a_1 - b_1 a_2}{a_1 - a_2} > 0.$$

Find a triangle of the maximal area with all inner vertices of the same colour.

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