# MA222 Metric Spaces

Course Program

Warwick University, 2018/2019

Part 1
- (a) Norms and metrics. Definitions and examples. Balls in different metrics. Metrics on spaces of sequences and spaces of functions.
- (b) Subspaces and product spaces (of metric spaces).
- (c) Open and closed sets, continuous functions.
- (d) Convergence of sequences in metric spaces and closed sets.
- (e) Equivalent metrics and isometries.

Part 2
- (a) Topology: definitions and examples. Topological properties vs. metric properties.
- (b) Subspaces and product spaces (of topological spaces)
- (c) Topological constructions: boundary, interior, closure. Cantor sets, $F_\sigma$, $G_\delta$, $F_{\sigma\delta}$ properties, dense, nowhere dense, and meagre sets.
- (d) Product, composition, and a linear combination of continuous functions.
- (e) Induced topology.
- (f) Metrizability, Hausdorff property and Zariski topology.
- (g) Normal topological spaces, Urysohn Lemma and Tietze Theorem.
- (h) Closure in metrizable and non-metrizable topological spaces.
- (i) Homeomorphsims and topological invariants.

Part 3
- (a) Compactness. Examples. Subspaces, intersections, unions and products of compact spaces. Tychonov's theorem.
- (b) Compactness and (semi)continuity
- (c) Compact sets in $\mathbb{R}^n$ and nets.
- (d) Uniform continuity.
- (e) Sequential compactness in metric spaces.
- (f) * Relative compactness.

Part 4
- (a) Connectedness. Examples. Connected subsets of $\mathbb{R}$.
- (b) Unions, products, and continuous images of connected sets. Connected components.
- (c) Path-connectedness.
- (d) Open sets in $\mathbb{R}^n$.
- (e) (Path)connectedness as a topological invariant.

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Part 5 (a) Completeness. Examples. Cauchy sequences. Completeness of $\mathbb{R}^n$.

(b) Complete and closed subspaces of a metric space.

(c) Completeness of the spaces of bounded and continuous functions.

(d) Banach fixed point Theorem and applications.

(e) Total boundedness. Definitions, properties, and a criterion.

(f) Compactness and total boundedness.

(g) Equicontinuity and uniform equicontinuity.

(h) Arzela-Ascoli Theorem and applications.

(i) Baire's category theorem and applications.

(j) Topological completedness and $G_\delta$ property.

(k) Completion: definition, existence, and examples.

(l) Compact metric spaces as imagages of the Cantor set. Peano curves.

(m) Baire classes.

The following books, jointly covering the course material, are available for download via the University Library:

1. Topology: an Introduction. S. Waldmann. Springer, 2014.
2. First Course in Metric Spaces. B. K. Tyagi. Foundation Books, 2012.
3. Elementary Theory of Metric Spaces: a Course in Constructing Mathematical Proofs. R. B. Reisel. Springer New York, 1982.
4. Metric Spaces. E.T. Copson. Cambridge University Press, 1968.

In addition, I can also recommend excellent synopsis by David Preiss (from 2015) and complete lecture notes by David Epstein (from 2000).