# A generalization of lemma on Triangle's Geometry

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In [1] N. I. Beluhov gave an elementary proof of Lester's theorem, using as lemma the following fact.

**Theorem.** In $\triangle APQ$ ($AP \neq AQ$) point $B$ is reflection of $P$ over $AQ$ and $C$ is reflection of $Q$ over $AP$. The tangent to $\odot(ABC)$ at $A$ intersect $PQ$ at $U$. Then reflection $T$ of $U$ over $A$ lies on $BC$.

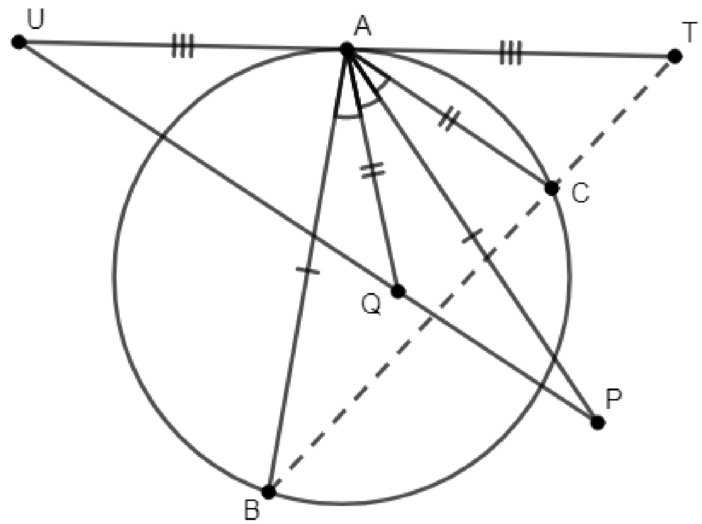


Figure 1: Original statement

Below we present the generalization, which has a very short and simple proof with an application of Desargues Involution Theorem.

# 1 Result

**Definition.** Division ratio $(XY;Z)$ of collinear points $X, Y, Z$ is such real number $r$, that $r\overrightarrow{ZY} = \overrightarrow{ZX}$.

**Theorem.** Given $\triangle ABC$ and points $P, Q$ such that $ABP \stackrel{-}{\sim} ACQ$. Denote by $\mathcal{H}$ homothety with center $A$ and ratio $k = (CQ; AP \cap CQ)$. The tangent to $\odot(ABC)$ at $A$ intersect $PQ$ at $U$. Then point $T = \mathcal{H}(U)$ lies on $BC$.

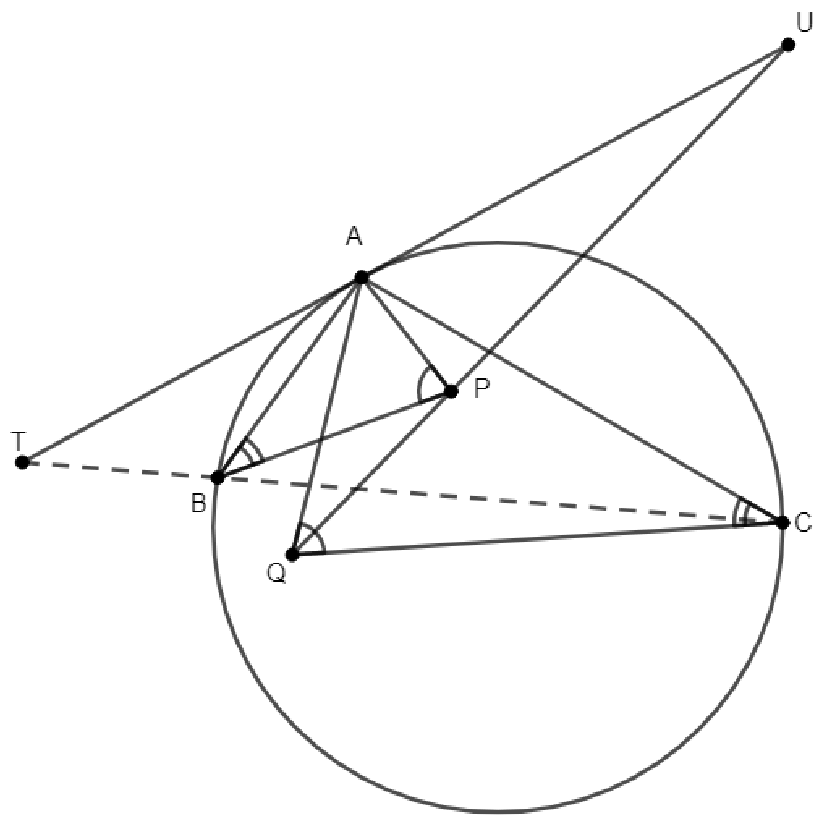


Figure 2: Generalization

# References

[1] N. I. Beluhov. An elementary proof of Lester's theorem. Journal of Classical Geometry, 1:53–56, 2012.