

A generalization of lemma on Triangle's Geometry

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In [1] N. I. Beluhov gave an elementary proof of Lester's theorem, using as lemma the following fact.

Theorem. In $\triangle APQ$ ($AP \neq AQ$) point B is reflection of P over AQ and C is reflection of Q over AP . The tangent to $\odot(ABC)$ at A intersect PQ at U . Then reflection T of U over A lies on BC .

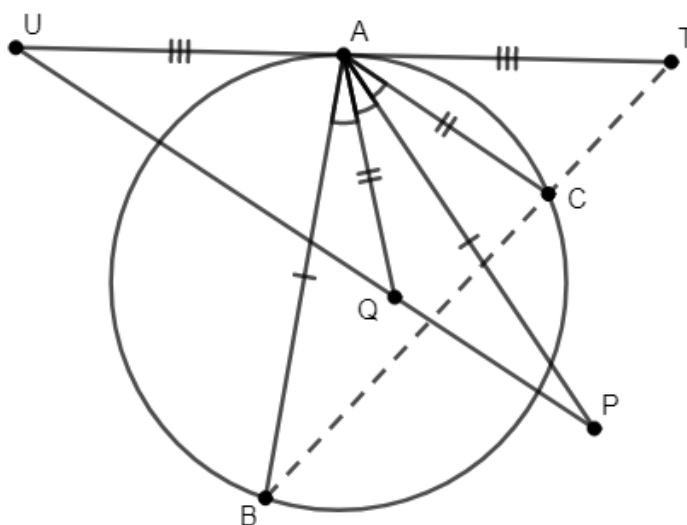


Figure 1: Original statement

Below we present the generalization, which has a very short and simple proof with an application of Desargues Involution Theorem.

1 Result

Definition. Division ratio $(XY; Z)$ of collinear points X, Y, Z is such real number r , that $r\overrightarrow{ZY} = \overrightarrow{ZX}$.

Theorem. Given $\triangle ABC$ and points P, Q such that $ABP \sim ACQ$ (triangles are similar with different orientation). Denote by \mathcal{H} homothety with center A and ratio $k = (CQ; AP \cap CQ)$. The tangent to $\odot(ABC)$ at A intersect PQ at U . Then point $T = \mathcal{H}(U)$ lies on BC .

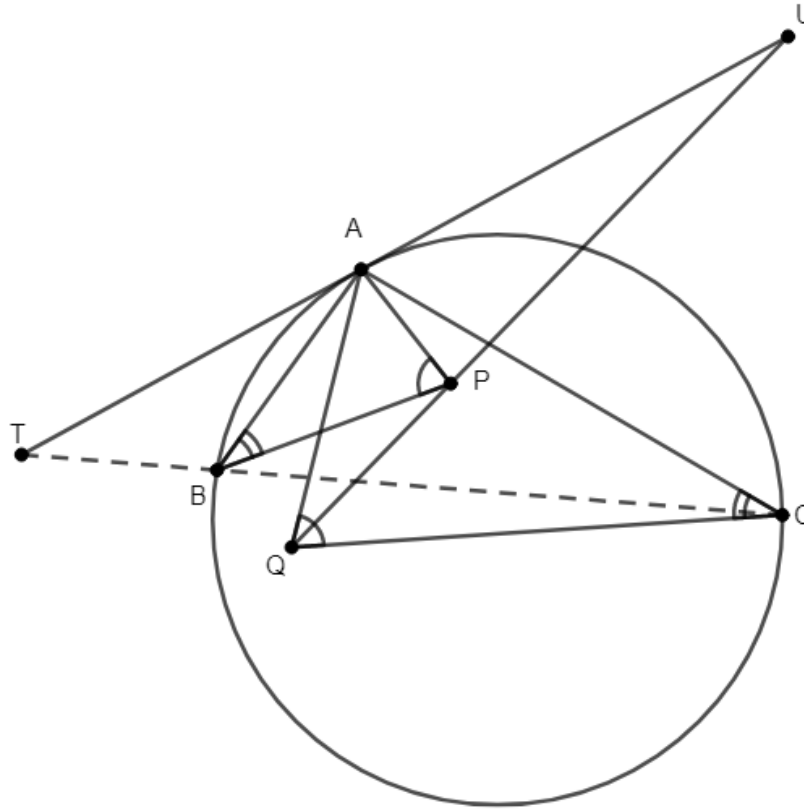


Figure 2: Generalization

References

- [1] N. I. Beluhov. An elementary proof of Lester's theorem. Journal of Classical Geometry, 1:53–56, 2012.