SUMMARY

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The main purpose of the research is the study of geometric objects using the associated DG (or A_∞) and triangulated categories, such as derived categories of coherent sheaves on algebraic varieties and Fukaya A_∞-categories of symplectic varieties.

1. *The homological mirror symmetry (HMS) conjecture.* HMS conjecture is a categorical interpretation of mirror symmetry. It states equivalence of certain categories associated with algebraic and symplectic sides. With the algebraic side, one associates the derived category of coherent sheaves (for varieties) or triangulated categories of singularities of singular fibers (for Landau-Ginzburg models). With the symplectic side one associates the Fukaya A_∞-category.

Here the following results were obtained by the author.:

1) a proof of Kontsevich-Soibelman conjecture which allows one to rigorously define Fukaya A_∞-category;

2) a proof of HMS conjecture for curves of higher genus treated symplectically;

3) a proof of HMS conjecture for punctured spheres treated symplectically (joint with A. Abouzaid, D. Auroux, L. Katzarkov and D. Orlov, in preparation).

The research here will focus on various special cases of HMS conjecture which eventually may lead to a proof in general case. In particular, the goal is to establish (the same as above direction of) HMS conjecture for punctured curves of genus $g \geq 1$.

2. *Exceptional collections.* Another subject is the study of (existence of) exceptional collections on varieties and stacks. There are a lot of smooth projective varieties and stacks X for which the derived category $D^b(X)$ is known to be generated by a full exceptional collection.

It was conjectured (originally by A. King) that every smooth nef-Fano toric DM stack (or at least each toric Fano variety) possesses a full strong exceptional collection of line bundles. Borisov and Hua have proved this conjecture in the case of Fano stacks for which either Picard number or dimension is at most two. The case of nef-Fano Del Pezzo stacks was further treated by Ishii and Ueda.

The author has disproved the conjecture by constructing infinitely many toric Fano varieties with Picard number three, which do not have full (not necessarily strong) exceptional collections of line bundles.

The research here will be focused on answering the questions of the following form:

Does a given variety or stack possesses a full (strong) exceptional collection (maybe of a certain type)?

In particular, the goal is to answer the following question: does each smooth projective toric DM stack possesses a strong exceptional collection.

3. Another goal is to prove or disprove the following conjecture of Kontsevich:

Conjecture 0.1. *Let X be a separated scheme of finite type over a perfect field. Let E be a generator of $D^b_{coh}(X)$, And $\mathcal{B}$ its endomorphism DG algebra. Then $\mathcal{B}$ is of homotopy finite type.*