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Summary

My research interests are mostly in the area of "Topological Homology", which is a version of relative homological algebra in categories of locally convex topological modules over topological algebras. This branch of mathematics lies at the intersection of Functional Analysis and Algebra, and it has the flavour of both subjects. I am also interested in relations between Topological Homology and some other fields of mathematics such as multivariable spectral theory and noncommutative geometry.

1. Past research. A *Fréchet algebra* is a complete, metrizable, locally convex $\mathbb{C}$-algebra. I proved that many classical Fréchet algebras do not have nonzero injective Fréchet modules. Among such algebras are algebras of smooth and holomorphic functions on real and complex manifolds and algebras of formal power series. Moreover, the injective homological dimension of nonzero Fréchet modules over such algebras can take only one (strictly positive) value.

To each Fréchet algebra A one may associate a number of invariants defined in homological terms. Among them are the *global dimension* $\mathrm{dg}\, A$, the *bidimension* $\mathrm{db}\, A$, the *weak global dimension* $\mathrm{w.dg}\, A$, and the *weak bidimension* $\mathrm{w.db}\, A$. I found these dimensions for some natural classes of Fréchet algebras. Concrete examples of such algebras include convolution algebras of smooth functions on compact Lie groups, algebras of "smooth compact operators" (which play an important rôle in K-theory and cyclic homology), various algebras of rapidly decreasing sequences, algebras of holomorphic functions endowed with the Hadamard product, smooth and complex analytic quantum tori. I also proved that, if A is a nuclear Fréchet–Arens–Michael algebra with $\mathrm{w.db}\, A < \infty$, then $\mathrm{w.dg}\, A = \mathrm{w.db}\, A$. In the "classical" homological algebra, a similar result seem to exist only for finite-dimensional algebras.

The *Arens-Michael envelope* of a $\mathbb{C}$-algebra A is the completion of A with respect to the family of all submultiplicative seminorms. This notion plays an important rôle in J. L. Taylor's noncommutative functional calculus; it is also potentially important for noncommutative geometry. I obtained explicit descriptions of the Arens–Michael envelopes for a number of standard noncommutative algebras, such as quantum affine spaces, quantum tori, quantum 2×2-matrices, and quantum Weyl algebras. For every algebra A from the above list, I proved that the embedding of A into its Arens–Michael envelope, B, is a *homological epimorphism*. Roughly, this means that all homological information about B-modules is preserved (and no new homological information appears) when we consider them as A-modules. I have also shown that if A is the universal enveloping algebra of a Lie algebra, then the standard embeddings of A into various complete topological algebras are homological epimorphisms.

2. Future research projects. My plan is to continue working in the same area and, in particular, to focus on some possible applications of topological algebras in noncommutative geometry. I am going to introduce *analytically finitely generated* (AFG) Fréchet algebras and to show that the category of commutative AFG algebras in anti-equivalent to the category of Stein spaces of finite embedding dimension. Thus AFG algebras can potentially be viewed as noncommutative analogs of algebras of holomorphic functions. I am going to study stability properties for the class of AFG algebras, to show that some concrete algebras that have been studied in the literature are in fact AFG, and to give a number of new examples. I also intend to study the exactness properties of a *noncommutative analytization functor* and to show that the functor is exact if the base algebra is "close to being commutative", but is not exact if the algebra is "highly noncommutative".

I also plan to calculate homological dimensions of some modules of holomorphic functions on closed submanifolds of Stein manifolds. The answer is supposed to depend on the existence (or nonexistence) of bounded plurisubharmonic functions on the manifold and on the submanifold.

Finally, I am going to develop a new general approach to (quantum) *Knapp–Vogan Hecke algebras*, which play an important rôle in representation theory of Lie groups and quantum groups. The proposed approach is supposed to be not only more general, but also more simple than the constructions of Knapp–Vogan (the classical case) and Sinel'shchikov–Stolin–Vaksman (the quantum case). The simplicity will be attained through an extensive use of multiplier Hopf algebra theory.