Summary of the project by A. Shlapunov
"The Cauchy Problem for Elliptic Complexes"

It is well known, that the Cauchy problem for elliptic equations is ill-posed in all the standard functional spaces. However it naturally appears in applications: in Hydrodynamics (as the Cauchy Problem for holomorphic functions), in Geophysics (as the Cauchy Problem for Laplace equation), in Elasticity Theory (as the Cauchy Problem for Lamé type systems), in Electrodynamics (as the Cauchy Problem for Maxwell system) and so on. Many papers were devoted to the problems, for instance see works of famous ones: J. Hadamard, T. Carleman, M.M. Lavrent'ev, V. Ivanov, V. Kondrat'ev, P. Lax, G. Fichera, and others.

At the end of XX-th century it was understood that the Cauchy problem for elliptic systems of linear equations is equivalent to another ill-posed problem: problem on "the analytic continuation" from a small open set to a bigger one. For overdetermined elliptic systems this approach was based on Integral Representation Method. It was elaborated with an active participation of L. Aizenberg, N. Tarkhanov, A. Shlapunov and others. The approach appeared also productive in both theoretical and practical ways: **simple formulae were constructed** for exact and approximate solutions to the problem.

Differential complexes appear as compatibility conditions for overdetermined systems of differential equations (such as, for example, Maxwell system or multidimensional Cauchy-Riemann system). One of the first problem of this kind was the Cauchy problem for the Dolbeault Complex (i.e. for the compatibility complex for the multidimensional Cauchy-Riemann operator), considered in papers by Andreotti and Hill. It has a great interest because of the famous example by Hans Lewy of a differential equation without solutions, constructed with the use of the tangential Cauchy-Riemann operator. Recently new approaches were found in a series of papers by such famous scientists as D.C. Hill, N. Tarkhanov, M. Nacinovich and others. Some original approaches to the problem were found by mathematicians of Krasnoyarsk School of Complex Analysis: A.M. Kytmanov, S. Myslivetz, A. Shlapunov and his students. For instance, A. Shlapunov described in his dissertation (of habilition) an interesting scheme for regularisation of the Cauchy problem for elliptic complexes, including operators of the same order, based on iteration methods.

We propose: 1) to apply the method above, reducing the Cauchy problem to the problem of "analytic continuation", **for study of the elliptic complexes**. Unfortunately, obstacles appear for the Dolbeault Complex already, because the operators, included to an elliptic complex, can be not elliptic. Many of them could be removed if we consider the problem in Lebesgue spaces. Hence we additionally need **to analyze boundary behaviour of distributions** of finite order of singularity: 2) to obtain complete results on **a regularisation of the Cauchy problem by iterational method**, described by A. Shlapunov in his dissertation (of habilitation). 3) Besides, a very perspective direction of the project might be **the study of non-linear Cauchy problem** (at least, for scalar elliptic operators), since recent approaches were mostly based on iteration methods and they worked under rather restrictive assumptions.

The main aim of the project is to obtain **solvability conditions and Carleman formulae** for exact and approximate solutions of the ill-posed Cauchy problem for linear and (in perspective) non-linear elliptic complexes.