Summary

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The theory of toric varieties plays an important role in modern Geometry, Algebra, Topology and Combinatorics. It is caused by a beautiful description of toric varieties in terms of convex geometry. Actual efforts to generalize such a description to other classes of objects are very natural.

The idea of toric varieties can be generalize in two various way. The first one is algebraical, the second one is combinatorial.

Algebraical way. It is also natural to replace the torus $\mathbb{T}$ with the commutative unipotent group $G_n = G_a \times \cdots \times G_a$, where G_a is the additive group of the ground field $\Bbbk$. The theory of generically transitive G_n-actions may be regarded as an "additive analogue" of toric geometry. In [13], an algebraic interpretation of generically transitive G_n -actions on $\mathbb{P}^n$ is given. Namely, it is shown there that such actions correspond to local finite-dimensional algebras. The paper [6] studies this correspondence in more details. Generically transitive G_n-actions on the non-degenerate quadric $Q_n \subset \mathbb{P}^{n+1}$ are described in [18]. It is shown there that for every n such action is unique up to isomorphism, which does not hold in general.

The first aim of the author is to work out a description of cubics $Q_n \subset \mathbb{P}^{n+1}$, which admit a unique additive structure.

A combinatorial way. Another definition of toric variety can be given in terms of fan, see [24, 12, 25]. A fan Δ is a set of polyhedral cones in vector space $\mathbb{R}^n$ glued along common faces. Bergman fans of matroids in tropical geometry has a familiar combinatorial structure.

One of the main goals of tropical geometry is to study classical algebraic geometry via polyhedral complexes. Tropicalisations of subvarieties of $(\mathbb{C}^*)^n$ are rational polyhedral complexes in R^n equipped with positive integer weights and satisfying the so-called balancing condition. For this reason tropical subvarities of $\mathbb{R}^n$ are considered to be polyhedral complexes with this added structure, [17, 15].

Before the advent of tropical geometry, Bergman fans were initially defined to be the logarithmic limit sets of complex algebraic varieties [9]. When equipped with appropriate weights they are tropical varieties in the above sense. For varieties defined by linear ideals, Sturmfels showed that the Bergman fan depends only on the underlying matroid. And, for simplicity, we call Bergman fan a matroidal fan.

Matroidal fans have many nice properties making them candidates for the local models of tropical non-singular spaces. Tropical linear spaces as studied by Speyer [21] and Speyer and Sturmfels [20] are locally matroidal fans. In addition, any codimension one cycle on a matroidal fan may be expressed as a tropical Cartier divisor, this is proved in [19].

The author turns his attention to automorphisms of matroidal fan. *The second aim of the author is to work out a description of automorphisms of Bergman fan in term of automorphisms of underlying matroid.*