Self-dual projective toric varieties

Projective duality can be thought as a way of recovering a projective variety from the set of its tangent hyperplanes. Dual varieties are a generalization to algebraic geometry of the Legendre transform of classical mechanics, and appear in several branches of mathematics. Generically, if $X \subset \mathbb{P}(V)$ is a projective variety, then its dual $X^\vee \subset \mathbb{P}(V^*)$ has codimension 1; if this is not the case, we say that $X$ is defective. A self-dual projective variety is a variety $X$ that is isomorphic to its dual as embedded varieties. Ein has provided the list of all smooth projective self-dual varieties; such a list contains all the smooth hypersufaces and a few more cases. In this talk we will focus on the duality of projective toric varieties; the study of the dual of these varieties has deep connections with the study of the so-called $A$-discriminants (Gelfand, Kapranov, Zelevinsky).

Let $T$ be an algebraic torus and $V$ a finite dimensional rational $T$-module. Let $X \subset \mathbb{P}(V)$ be the closure of a $T$-orbit in $\mathbb{P}(V)$ — a projective toric variety. We will classify in terms of the combinatorial data associated to such a variety, when $X$ is self-dual. In this way, we provide a large family of self-dual projective varieties. This is a joint work with M. Bourel and A. Dickenstein.