A Stay-in-a-Set Game without a Stationary Equilibrium

Kristoffer Arnsfelt Hansen Mikhail Raskin, raskin@mccme.ru<sup>1</sup>

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- Turn-based (perfect information)
- Finite
- Stochastic
- Independent safety goals

## Exact Nash equilibrium exists with bounded memory

Remember who has lost

Induction

- 1 player: Markov decision process
- Arbitrary positional strategy for each player in case of loss
- Uniform bound on time until someone loses or loss becomes impossible
- Payoffs at that point: by inductive assumption (or obvious)

- For 2 players, is there always **zero** memory Nash equilibrium (instead of 2 bits of memory)?
- (i.e. equilibrium in stationary strategies)

Discounted payoffs: stationary equilibrium Deterministic game: stationary equilibrium Recursive/imperfect information games: no stationary  $\varepsilon$ -Nash equilibrium

## Example without stationary equilibrium



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## If cycle is left, both P1 and P2 prefer that P1 leaves For P1 best case is staying For P2 staying means loss

Note that  $\varepsilon$ -equilibrium exists

(for two players)

Stationary  $\varepsilon$ -equilibrium? Stationary equilibrium for reachability conditions? Stationary equilibrium for deterministic recursive games with reachability conditions?

## Thanks for your attention!

Questions?

Discounted payoffs: stationary equilibrium Deterministic game: stationary equilibrium Recursive/imperfect information games: no stationary  $\varepsilon$ -Nash equilibrium

Two-player turn-based stay-in-a-set game: no stationary equilibrium

Stationary  $\varepsilon$ -equilibrium? Stationary equilibrium for reachability conditions? Stationary equilibrium for deterministic recursive games with reachability conditions?