Efficient Restrictions of Immediate Observation Petri Nets

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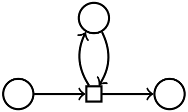
Outline

- General IO nets
- Non-forgetting IO nets
- No-near-miss marking pairs
- Conclusion

IO nets

Immediate observation Petri nets

- In-degree and out-degree two (Population protocol)
- Pre-set and post-set have intersection (Observed place)



IO nets: reachability complexity

- PSPACE-complete [Esparza, R., W.-K.: 2019]
- Hardness construction:
  Turing machine modelling
  Few tokens, enabling/disabling transitions many times
- PSPACE inclusion proof:
  Most situations much easier
- Interesting simpler restrictions?

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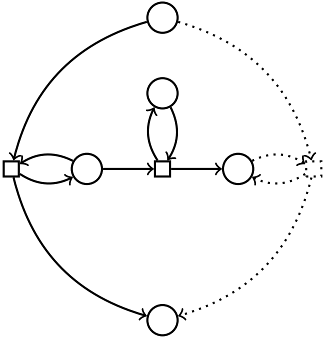

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- Similar to DO [Angluin et al.: 2007]

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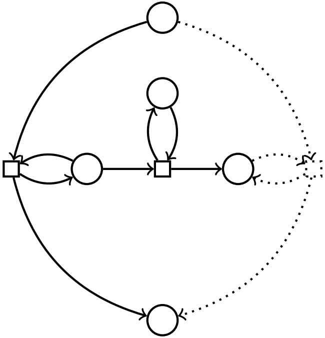

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Non-forgetting IO nets: reachability complexity

Theorem

Reachability in non-forgetting IO nets is NP-complete

Feasibility:

- Move = class of equivalence of transitions
  under observation replacement
- Guess the order of moves getting enabled
- Integer maximum flow for fixed set of moves
- Greedy reconstruction of firing sequence

Hardness:

- Reduction from CIRCUIT-SAT
- Simulate evaluation
- Guess inputs

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No-near-miss marking pairs

Motivation: large systems

Chemical system: 10^9 is a small number

… and 10^{-6} precision means exact equality

Let X and Y be sets of states, let M and M' be markings

$M(X) \approx M'(Y)$ implies conservation law… and $M(X) = M'(Y)$

No near misses:

$$M(X) \approx M'(Y) \overset{\text{def}}{\Leftrightarrow} |M(X) - M'(Y)| \leq |P|^3$$

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Reachability for no-near-miss marking pairs is polynomial.
Moreover, in polynomial time one can do one of the following:

- *correctly report unreachability,*
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No-near-miss marking pairs: reachability complexity

Proof ideas:

- Track possibilities «from p to q via r»
- Stable set of restrictions:
  Consistent with reachability via token moves
  Flow exists using possible paths
- Maximum flow with large flow over each used arc
  Failure to build yields near miss
- Uniformly large flow can be converted to firing sequence

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- From approximately M to approximately M'
- Conjecture: PSPACE-complete to verify reachability between two neighbourhoods
  even for $|M| = |M'|$
- Conjecture: polynomial algorithm for gap problem
  either impossibility for smaller neighbourhoods
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- Work in progress on both

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Conclusion

- IO net reachability PSPACE-hardness not very robust
- Simpler restrictions that can be useful
  - Non-forgetting nets *NP-complete*
  - No-near-miss marking pairs *P*
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Thanks for your attention!

Questions?