APPLICATION SUMMARY by MIKHAIL SKOPENKOV

Research is planned in 3 almost independent directions. The main direction is *discrete complex analysis*, additional directions are *rational classification of embeddings* and *classification of circular surfaces*.

I. Discrete complex analysis. Various constructions of complex analysis on planar graphs were introduced by Isaacs, Duffin, Mercat, Dynnikov–Novikov, Bobenko–Mercat–Suris, Bobenko–Pinkall–Springborn. Recently this subject is developed extensively due to applications to statistical physics (Chelkak–Smirnov), numerical analysis (Hilderbrandt et al.), computer graphics (Mercat), combinatorial geometry (Kenyon).

One of our past results was application of discrete complex analysis to description of all polygons which can be tiled by squares (with Prasolov, 2010). This was known earlier only for a rectangle (Dehn) and for an L-shaped hexagon (Kenyon). Our result solved a problem of Freiling et al. from 2000.

The first planned result is convergence of discrete harmonic functions on general lattices to a harmonic function when the lattice becomes finer and finer. This was proved earlier only for square lattices by Courant–Friedrichs–Lewy and for rhombic lattices by Chelkak–Smirnov and implicitly by Ciarlet–Raviart. Our result solves a problem of Smirnov from 2010. It was presented at the seminars of Bobenko, Sinai, Smirnov.

Let us give precise statement. A *quadrilateral lattice* is a graph $Q \subset \mathbb{C}$ with rectilinear edges such that each bounded face is a quadrilateral. Depending of the shape of faces, one speaks about *square*, *rhombic*, or *orthogonal* lattices (the latter are quadrilateral lattices such that the diagonals of each face are orthogonal). A function $f\colon Q \to \mathbb{C}$ is *discrete analytic*, if $\frac{f(z_1)-f(z_3)}{z_1-z_3} = \frac{f(z_2)-f(z_4)}{z_2-z_4}$ for each quadrilateral face $z_1z_2z_3z_4$ of Q. The real part of a discrete analytic function is called a *discrete harmonic function*. Given a function $u\colon \mathbb{C} \to \mathbb{R}$, the *Dirichlet problem on* Q is to find a discrete harmonic function $f_{Q,u}\colon Q \to \mathbb{R}$ such that $f_{Q,u}(z) = u(z)$ for each vertex $z \in \partial Q$. We omit the technical definition of a *nondegenerate uniform sequence of lattices approximating a domain* $\Omega \subset \mathbb{C}$.

Theorem. *Let $\Omega \subset \mathbb{C}$ be a domain bounded by a smooth closed curve $\partial\Omega$ without self-intersections. Let $u\colon \mathbb{C} \to \mathbb{R}$ be a smooth function. Take a nondegenerate uniform sequence of finite orthogonal lattices $\{Q_n\}$ approximating the domain Ω. Then the solution $f_{Q_n,u}\colon Q_n \to \mathbb{R}$ of the Dirichlet problem on Q_n converges to the solution $f_{\Omega,u}\colon \Omega \to \mathbb{R}$ of the Dirichlet problem in Ω (with the boundary values $u\,|_{\partial\Omega}$) uniformly on any compact subset of Ω.*

It is also planned to advance in the proof of the following assertions:

- uniform convergence of the Galerkin finite element method on planar Delauney triangulations;
- convergence of the random walk on general (in particular, random) planar graphs to the Brownian motion;
- convergence of period matrices on discrete Riemannian surfaces to their continuous counterparts.

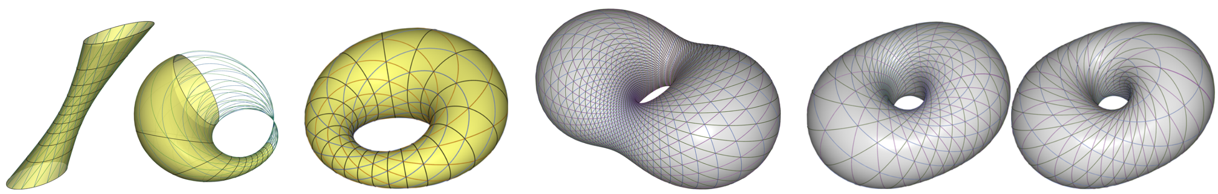
Features of the roposed new approach to discrete complex analysis are applications of *alternating-current networks*, *energy estimates*, and techniques of *electrical impedance tomography*.

II. Rational classification of embeddings. Classification of embeddings of manifolds is a classical problem of topology. In general one can hope only to reduce it to problems of homotopy theory, and thus to obtain either quantitative results in particular cases or qualitative results in the general case. Our aim is to determine, *if the set of isotopy classes of embeddings $N \to S^m$ is finite* for simplest manifolds N:

- disjoint unions of spheres, not necessarily of the same dimension (with Crowley and Ferry, accepted);
- products of two spheres, in the so-called 2-*metastable* dimension (with Cencelj and Repovš, accepted);
- products of two spheres, under very weak dimension restriction (submitted).

III. Classification of circular surfaces. Surfaces generated by simplest curves (lines and circles) are popular subject in pure mathematics and have applications to design and architecture. Our aim is to classify:

- all ruled Laguerre minimal surfaces (with Pottmann and Grohs, accepted);
- all surfaces containing both a line and a circle through each point (with Nilov, submitted);
- all surfaces containing at least two circles through each point (with Pottmann and Krasauskas);
- all *webs*, i.e., triangulations of surfaces by circles; see the figure (with Pottmann and Shi, submitted).



Teaching. It is planned to continue the following past activities:

- teaching assistance at Independent Moscow University (half-year graduate courses);
- informal supervision of undergraduate students in form of writing joint research papers;
- organization and teaching at distance mathematical school for olympiad winners;
- teaching at summer schools for high school students;
- writing popular science papers.