Research Statement of Ivan Arzhantsev: Summary

My research interests lie in algebraic geometry and representation theory. I study algebraic transformation groups, toric varieties, linear algebraic groups and their representations, invariant theory, Lie groups and Lie algebras. Specific subjects are algebraic homogeneous spaces and their embeddings, Cox rings, T-varieties, combinatorial methods of geometric invariant theory, and automorphisms of algebraic varieties.

Cox rings extend in a universal manner the concept of homogeneous coordinates from projective spaces to a large class of varieties. Our current book project provides an introduction to this active research area in the field of algebraic and arithmetic geometry.

Study of algebraic varieties equipped with a regular action of an algebraic torus T is a classical topic in algebraic geometry. Recently a semi-combinatorial description of T-varieties in terms of so-called polyhedral divisors and polyhedral fans was introduced by Klaus Altmann and Juergen Hausen. It is an important problem to generalize this approach to certain classes of actions of arbitrary reductive groups. The most reasonable class form actions with spherical or even horospherical orbits.

In his seminal article Michel Demazure gave a description of the automorphism group of a smooth complete toric variety. Later David Cox reproved and generalized this result in terms of total coordinates. Our aim is to obtain a similar description for the automorphism group of a complete T-variety of small complexity in terms of Cox rings.

Another attractive problem is a description of automorphisms of non-complete toric varieties. Recent proof of Nagata's conjecture on wild automorphisms of the affine 3-space by Ivan Shestakov and Ualbai Umirbaev is a great contribution to the study of automorphisms of affine spaces. At the same time the automorphism group of the affine 3-space is very far from being well understood. Cox rings allow to extend notions of tame and wild automorphisms to arbitrary affine toric varieties. We plan to look for further examples of wild automorphisms of affine threefolds.

Recently we classified acyclic curves on affine toric surfaces generalizing famous Abhankar-Moh-Suzuki and Lin-Zaidenberg Theorems for the affine plane. Such a classification leads to a description of the automorphism group of an affine toric surface as a free amalgamated product similar to the Jung – van der Kulk Theorem.

An affine variety X is flexible if the tangent space at any smooth point on X is generated by sections of locally nilpotent vector fields. Flexibility turns out to be equivalent to the condition that the action of the group of special automorphisms on the smooth locus of X is infinitely transitive. Non-degenerate affine toric varieties, affine cones over flag varieties and suspensions over flexible varieties are flexible. The next task is to check the flexibility condition for affine spherical varieties and for special classes of T-varieties.

A complete toric variety is an equivariant completion of an algebraic torus T. Replacing T by a commutative unipotent group $\mathbb{G}_a^n$ one may look for an "additive analogue" of toric geometry. A remarkable correspondence between generically transitive $\mathbb{G}_a^n$-actions on projective spaces and Artinian local algebras was established by Brendan Hassett and Yuri Tschinkel. We develop this correspondence and classify all such actions of modality one as well as actions on projective hypersurfaces. It is planned to extent Hassett-Tschinkel's correspondence to arbitrary commutative linear algebraic groups and arbitrary Artinian algebras. This will provide a framework to study equivariat completions of commutative algebraic groups.