REPORT ON THE DINASTY–IUM FELLOWSHIP 2016

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Results 2014-2016

- We prove the conjecture by Feigin, Fuchs and Gelfand describing the Lie algebra cohomology of formal vector fields on an n-dimensional space with coefficients in symmetric powers of the coadjoint representation. We also compute the cohomology of the Lie algebra of formal vector fields that preserve a given flag at the origin. The latter encodes characteristic classes of flags of foliations and was used in the formulation of the local Riemann-Roch Theorem by Feigin and Tsygan.

  Feigin, Fuchs and Gelfand described the first symmetric power dealing with quite complicated computation in invariant theory. We apply different degeneration theorems of appropriate Hochschild-Serre spectral sequences and get the description of all symmetric powers at once without many computations.
- While working with an odd analogue of a Lie bialgebra (the parities of Lie bracket and coLie cobracket is supposed to be different) we introduce a natural notion of an involutive odd Lie bialgebra, and, moreover, we construct a small resolution of the corresponding properad what follows the good quantization properties for representations of the corresponding properad.
- The little cubes operad E_n governs natural operations one has on the iterated loop spaces. This operad is the most popular, however, we are far from understanding it in full details. In particular, it was proved recently by F.Brown that the set of homotopy infinitesimal automorphisms of this operad contains a free Lie algebra with infinite amount of generators and conjecturely (Deligne-Drinfeld conjecture) this coincides with the whole set of zero deformations. However, so far we do not know any reasonable conjecture about the description of cohomology classes of higher degrees of the deformation complex of the little cubes operad. One of the model of this deformation complex has a simple combinatorial description and is called Kontsevich's graph complex. In a series of joint (with T. Willwacher and M. Zivkovic) papers "Differentials on graph complexes", "Differentials on graph complexes II - hairy graphs" we invent new transgressive differentials (for both even and odd cases) and use that to prove the infinite-dimensionality of higher deformations of E_n and the cohomology of the deformation complex for the map between little cubes operads of different dimensions. Moreover, we compute some particular classes in small degrees and derive different bounds on Betti numbers. The problem of the description of the homotopy deformations of the map $E_n \to E_m$ governs the description of the rational homotopy type of the space of embedding $\mathbb{R}^m \hookrightarrow \mathbb{R}^n$ after Goodwillie–Weiss.

  We also announced in the project to derive a description of the homotopy type of the framed little discs operad. Unfortunately, the corresponding results are still contained in an unpublished manuscript (joint with T. Willwacher and V. Turchin)

which suprisingly contains a lot of tehnical details that we are currently trying to decrease.

- In a series of papers on the relationships of Macdonald polynomials and BGG properties for the category of graded representations of a Lie algebra of $\mathfrak{g}$-valued currents we proved several results which were announced in the project. In particular, we proved the equivalence of the BGG properties for the category of representations and the orthogonality of characters of Weyl modules, what follows that for a simple Lie algebra $\mathfrak{g}$ the characters of local Weyl modules coincide with q-Hermite polynomials. Moreover, we categorify the Macdonald pairing and defined complexes of graded modules over super Lie algebra of currents $\mathfrak{g} \otimes \mathbb{C}[x, \xi]$ whose equivariant Euler characteristic coincides with Macdonald polynimials.

Papers 2014-2016

(1) "Highest weight categories and Macdonald polynomials"
Preprint available at math.arxiv.org:1312.7053, 33pp.
(2) "Differentials on graph complexes II - hairy graphs" (with T. Willwacher and M. Živković)
Preprint available at math.arxiv.org:1508.01281.
(3) "On Quantizable Odd Lie Bialgebras" (with S. Merkulov and T. Willwacher)
Letters in Mathematical Physics, (2016) 106(9), 1199–1215
(4) "Differentials on graph complexes" (with T. Willwacher and M. Živković)
accepted by Advances in Mathematics (2016);
(5) "Characteristic classes of flags of foliations and Lie algebra cohomology"
Transformation groups, Volume 21, (2016), Issue 2, pp 479–518
(6) "On generating series of finitely presented operads" (with D. Piontkovski),
Jounal of Algebra, Volume 426, 2015, pp. 377–429
(7) "Macdonald Polynomials and BGG reciprocity for current algebras"
(with A. Berenstein, M. Bennet, V. Chari, S. Loktev) //
Selecta Mathematica New series, April 2014, Volume 20, Issue 2, pp 585–607

Scientific conferences and seminar talks (2016)

Talk "On categorification of Macdonald polynomials"
at *Seminaire on mathematical physics*, NIU HSE Moscow;
Talk "Compactified Moduli spaces of curves as a Homotopy quotients of operads"
at *Characteristic classes and intersection theory* seminar, NIU HSE Moscow;

Teaching (2016)

[1] Quantum Groups II. Independent University of Moscow, III year and higher level students, February-May 2015, 4 hours per week (2 hours lecture + 2 hours seminar).

This is the second part of the course which wants to clarify the notions and ideas invented by V.Drinfeld developed in the series of papers on quantum groups.

program:
- Coalgebras, Hopf algebras and tensor categories;
- Quantization and classical limit, Poisson algebras;
- Poisson-Lie groups, Lie bialgebras;
- Coboundary, triangular and quasitriangular Lie bialgebras;
- Drinfeld Double and universal R-matrix;
- Operad theory and Tamarkin's quantization;

- Little discs operad and Deligne's conjecture;
- Braided tensor categories;
- KZ-equations and Drinfeld category;

[2] Algebraic theory of D-modules. Higher School of Economics, Fall 2016, faculty of mathematics, III year and higher level students, 3 hours per week (2 hours lecture + 1 hours seminar)

program:

Analitic continuation and Bernstein-Sato polynomials;
Gelfand-Kirillov dimension;
Holonomic modules;
Singular support, singular cycles and Habbers theorems;
Homological properties of functional dimension;
Sheaf of differential operators and D-modules on general algebrac;
Kashiwara's theorem;
Functors on D-modules;
D-affine property of projective spaces;
Perverse sheaves and Riemann-Hilbert correspondanc;
Kazhdan-Lustig polynomials.

[3] Algebra-I (Fall 2016) Higher School of Economics, faculty of mathematics, I year undergraduate students (4 hours lectures + 2 hours seminar per week)

[4] Galois theory (Spring 2016) Higher School of Economics, faculty of mathematics, II year undergraduate students (3 hours seminars per week)

[5] Geometry (Fall 2016) Higher School of Economics, faculty of mathematics, I year undergraduate students (2 hours seminars per week)