A REPORT OF A. SKOPENKOV FOR THE DYNASTY FOUNDATION GRANT, 2014

1. Research results

We work in the smooth category. For a smooth manifold N denote by $E^m(N)$ the set of smooth embeddings $N \to \mathbb{R}^m$ up to smooth isotopy.

In [3] for $p \le q$ and $m \ge 2p + q + 3$ I construct a group structure on $E^m(S^p \times S^q)$, and describe this group in terms of homotopy groups of spheres and easier-to-calculate groups of embeddings. Earlier such a description was known only for $2m \ge 3p + 3q + 4$. I use a recent exact sequence of M. Skopenkov. See more details in my proposal.

Let N be a closed connected orientable 4-manifold with torsion free integral homology. The main result of [4] is *a complete readily calculable classification of embeddings* $N \to R^7$. This is done

• in the 'smooth modulo knots' category, i.e. we describe the set $E^m(N)/\#$ of smooth embeddings $N \to R^7$ up to smooth isotopy and connected sums with smooth embeddings $S^4 \to S^7$.

• in the piecewise-linear (PL) category, i.e. PL embeddings $N \to R^7$ up to PL isotopy are classified.

Such a classification was earlier known only for simply-connected N, in the PL case by Boéchat-Haefilger-Hudson 1970, in the smooth case by the authors 2008. In particular, for $N = S^1 \times S^3$ we define geometrically a 1–1 correspondence between the set of PL isotopy classes of PL embeddings $S^1 \times S^3 \to R^7$ and the quotient set of $Z \oplus Z_6$ up to equivalence $(l, b) \sim (l, b')$ for $b \equiv b' \mod 2GCD(3, l)$. See more details in my proposal.

The paper [2] originated from the following problem was suggested by E. Rees in 2002: describe the action of self-diffeomorphisms of $S^p \times S^{n-p}$ on $E^m(S^p \times S^{n-p})$.

Let $g : S^p \times S^{n-p} \to R^m$ be an embedding such that $g|_{a \times S^{n-p}} : a \times S^{n-p} \to R^m - g(b \times S^{n-p})$ is null-homotopic for some different points $a, b \in S^p$ and $m \ge n + 2 + \frac{1}{2}\max\{p, n-p\}$.

Theorem. *For a map $\varphi : S^p \to SO_{n-p}$ define an autodiffeomorphism φ' of $S^p \times D^{n-p}$ by $\overline{\varphi}(a, b) := (a, \varphi(a)b)$. Let φ'' be the S^{n-p-1}-symmetric extension of φ to an autodiffeomorphism of $S^p \times S^{n-p}$. Then for each map $\varphi : S^p \to SO_{n-p}$ embedding $g \circ \varphi''$ is isotopic to embedded connected sum $g\#u$ for some embedding $u : S^n \to S^m$.*

Let N be an oriented n-manifold and $f : N \to R^m$, $g : S^p \times S^{n-p} \to R^m$ are embedding. As a corollary we obtain the following result on S^p-parametric embedded connected sum $f\#_s g$ (defined earlier by the author).

Under certain conditions for orientation-preserving embeddings $s : S^p \times D^{n-p} \to N$ the 'smooth modulo knots' class in $E^m(N)/\#$ of $f\#_s g$ depends only on f, g and the isotopy (the homotopy or the homology) class of $s|_{S^p \times 0}$.

See more details in my proposal.

2a. Research papers

[1] D. Gonçalves and A. Skopenkov, A useful lemma on equivariant maps, Homology, Homotopy and Applications, 16:2 (2014), 307 - 309.

[2] A. Skopenkov, How do autodiffeomorphisms act on embeddings, submitted, http://arxiv.org/abs/1402.1853

[3] A. Skopenkov, Classification of knotted tori, preprint, 2014,

[4] D. Crowley and A. Skopenkov, Classification of smooth embeddings of non-simply-connected 4-manifolds into R^7, preprint, 2014

2b. Expository publications for university students.

[5] A. Chernov, A. Daynyak, A. Glibichuk, M. Ilyinskiy, A. Kupavskiy, A. Raigorodskiy and A. Skopenkov, Elements of Discrete Mathematics As a Sequence of Problems, Moscow, MCCME, to appear, http://www.mccme.ru/circles/oim/discrbook.pdf

[6] A. Skopenkov, Algebraic Topology from Geometric Viewpoint, http://arxiv.org/abs/0808.1395 v2, http://www.mccme.ru/circles/oim/obstruct.pdf (a new version prepared) Moscow, MCCME, to appear

[7] D. Ilyinskiy, A. Raigorodskiy and A. Skopenkov, Existence proofs in combinatorics using independence, Mat. Prosveschenie, 19 (2015), http://arxiv.org/abs/1411.3171

[8] V.V. Prasolov, A.B. Skopenkov, Some reflections on why Lobachevsky geometry was recognized, Mat. Prosveschenie, 19 (2015), http://arxiv.org/abs/1307.4902 (a new version prepared)

[9] A. Skopenkov, Some more proofs from the Book: solvability and insolvability of equations in radicals, in Russian, submitted, http://arxiv.org/abs/0804.4357 v6 (a new version prepared)

[10] A. Skopenkov and M. Skopenkov, Some short proofs of the unrealizability of hypergraphs, http://arxiv.org/abs/1402.0658, submitted.

3. Conferences

International conference on embeddeed graphs, October, 2014. Talk “Classification of link maps”.

Conference of Moscow Institute of Physics and Technology, November, 2014. Talk “Basic embeddings and Hilbert’s 13th problem”.

(I was unable to attend some conferences because of a broken leg.)

4. Work in scientific centers and university groups

I worked in Moscow Mathematical School. In particular, I delivered talks at

• Topology Seminar, Faculty of Mathematics, Higher School of Economics, talk “How do autodiffeomorphisms act on embeddings”.

• Topology Seminar, Steklov Mathematical Institute, talks “Classification of knotted tori”.

• Postnikov memorial seminar, Moscow State University, talk “How do autodiffeomorphisms act on embeddings”.

• Seminar of N.P. Dolbilin and N.M. Moschevitin, Moscow State University, talk “Embedding and knotting of manifolds in Euclidean spaces”.

• Seminar of A. M. Raigorodskiy, Moscow State University, talk “Embedding and knotting of manifolds in Euclidean spaces”.

I continued collaboration with D. Crowley from Univ. of Bonn / Univ. of Edinburgh and with D. Gonçalves from Univ. of Sao Paolo.

5. Teaching

[1] Combinatorial topology, II year students, February-May 2014, 2 hours per week. Moscow Institute of Physics and Technology (DIHT)

[2] Modern topological methods in physics, II year students, February-May 2014, 2 hours per week. Moscow Institute of Physics and Technology (DGAP)

[3] Knot Theory, IUM, MiM, February-May 2014, 4 hours per week.

[4] Topology-I, I year students, Independent University of Moscow, February-May 2014, 4 hours per week (joint with M. Skopenkov).

[5] Topology-II, II year students, Independent University of Moscow, September-December 2014, 4 hours per week (joint with M. Skopenkov).

[6] Discrete analysis (exercises), II year students, February-December 2014, 2 hours per week. Moscow Institute of Physics and Technology

[7] Vector fields on 2- and 3-manifolds (minicourse), Summer School ‘Modern Mathematics’, July 2014, 8 hours.

I was an advisor of papers by S. Avvakumov (Master thesis, http://arxiv.org/abs/1408.3918) D. Akhtyamov (http://arxiv.org/abs/1411.4990) and D. Kolodzey (premaster work).