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Homogenization of the Neumann's brush problem

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Homogenization occurs in partial differential equations when one considers a sequence of equations which describe very heterogeneous media. I will begin by recalling two classical cases of such a situation, the case where the coefficients widely vary because the body is made of a mixture or different materials, and the case where the domain is perforated by a lot of small holes with Dirichlet's boundary condition on this very fragmented boundary, and I will present in each case the result of this process.

After this introduction, I will specifically consider another problem of the same family: the problem of the Neumann's brush. This is the case of a domain which has the form of a brush (in dimension N = 3) or of a comb (in dimension N = 2), i.e. which is composed of cylindrical vertical teeth distributed over a fixed basis. All the teeth have the same fixed height, but their cross sections can vary from one teeth to another one and the teeth can be adjacent, i.e. they can share parts of their boundaries. The diameter of every tooth is supposed to be less than or equal to some parameter epsilon which tends to zero, and the asymptotic volume fraction of the teeth is supposed to be bounded from below away from zero. No periodicity is assumed on the distribution of the teeth. In this widely varying domain one studies the asymptotic behavior of heath conduction, namely the solution of the Laplace equation with a zeroth order term, when the Neumann boundary condition is imposed on the whole of this complicated boundary. I will revisit this problem in the light of a recent work of Antonio Gaudiello (Naples, Italy), Olivier Guibe (Rouen, France), and myself, explaining how the transmission of the heat behaves in the teeth when the source term belongs to L^2. This is a classical problem but our homogenization result takes place in a geometry which is more general that the ones which were considered before. Moreover, we obtain a corrector result which is new. This is proved by using a very simple test function. Finally, if time permits, I will consider the case where the source term belongs to L^1, which motivated our work. Working in the framework of renormalized solutions, and introducing a definition of renormalized solutions for degenerate elliptic equations where only the vertical derivative is involved (such a definition is new), we are able to identify the limit problem and to prove a corrector result.