"Young Russian Mathematics" award
Scientific report for 2017
Denis Volk

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1. Scientific Results and Papers

1.1. Dynamics of Piecewise Translations

Piecewise translations (PWTs) and a wider class of maps, piecewise isometries, have many applications in computer science, machine learning and electrical engineering, see [1]. In dimension 1 invertible PWTs which preserve Lebesgue measure are interval exchange transformations (IETs) which are classic objects in ergodic theory but still attract a lot of interest. They have deep connections with polygon billiard maps, measurable foliations, translation flows, Abelian differentials, Teichmüller flows and other areas.

Boshernitzan and Kornfeld who were the first to consider general PWTs in dimension 1, showed that ITMs of rank less than 2 (i.e. that the endpoints and translation vectors of an ITM span a 2-dimensional subspace over the rational numbers) are finite type. This means that is their dynamics reduce to the attractor after finitely many iterations. Then the dynamics on the attractor is a disjoint union of finitely many IETs. This allows to apply classic results such as unique ergodicity and weak mixing of almost every IET. They also provided an example of a rank 3 ITM of infinite type and questioned how typical are infinite type ITMs. In 2014, we demonstrated that almost every ITM of three intervals is finite type [2] .

The study of piecewise isometries in 2 or more dimensions is still in its relative infancy. Recently we could do one of the first steps towards understanding of general PWTs in arbitrary dimension. Namely, we could show that *any* piecewise translation with $m = d + 1$ pieces in $\mathbb{R}^d$ with rationally independent translation vectors is finite type. The result is presented in recent preprint [1] along with some other results of ergodic type.

1.2. Cross-frequency interactions in human brain

In collaboration with Vadim Nikulin (Max Planck Institute for Human Cognitive and Brain Sciences, Leipzig, and HSE, Moscow) and two Master students, Igor Dubinin (MIPT, Moscow) and Alexandra Myasnikova (HSE, Moscow), we carried out a project in which we developed a new method for finding synchronized activity in human brain cortex based electro- and magnetoencephalography (E/MEG) recordings.

In 2012, Nikulin et al. [3] proposed a novel method for finding cross-frequency phase synchronized components in multi-channel E/MEG signal. This method could treat only integer frequency ratios, i.e., $f_1 : f_2 = 1 : q$. We developed a new mathematical model which, in particular, extends the range of uses to any rationally related frequency bands f_1 and f_2, $f_1 : f_2 = p : q$, p, q are integers. In practice, the numbers p, q in biological resonances $p : q$ are typically within $1 \le p, q \le 5$. Numerical experiments with simulated EEG signals show that the new method works extremely reliably in this p, q range. For $p = 1$, the core idea of the new method reduces to the one used in [3] but we also made several improvements on the top of this.

We developed a MATLAB toolbox which allows to apply the new method to E/MEG signals from behavioral, cognitive or any other experiments. The open source code is available at
https://bitbucket.org/dsvolk/rhythms-xfreq

The algorithm is well-tested on simulated data and ready to use. We are currently implementing a few more optimizations.

We applied the algorithm to data from a Steady State Visual Evoked Potential experiment. The analysis revealed some interesting functional interactions in human brain which were previously unknown. We are preparing a paper with the description of the algorithm, results of simulation tests and analysis of real data.

2. New Projects and Collaborations

In Fall 2017, I joined Interdisciplinary Scientific Center J.-V. Poncelet, Moscow, to work with Sergey Nechaev on a project joint between mathematics and neuroscience. The mathematical part of the project is concerned with some special big random graphs and spectra of their Laplace operators. The neuroscience part of the project is about general understanding what are physical reasons and substrates for consciousness. The aim of the project is to build a bridge between these two areas of science and to transfer understanding in both directions.

3. Participation in Schools and Conferences

In April, I visited Max Planck Institute for Human Cognitive and Brain Sciences, Leipzig, for collaboration with Vadim Nikulin.

In May, I participated in a conference "Brain stimulation", Moscow, in particular, the talks which related brain excitatory responses to stimulation to the structure of underlying connectomics graph.

In June, I participated in a conference "Cognitive Science in Moscow" and gave series of talks at summer school "Dynamical Systems", Stola, Slovakia, organized by Yu. S. Ilyashenko.

In July, I gave a course "Mathematics of brain: mathematical models in neuroscience" at summer school "Contemporary Mathematics 2017", Dubna, Russia.

In August, I gave an invited talk on conference "Just a little computation in dynamics", Bedlewo, Poland. The talk was about our recent advances in understanding piecewise translation maps in higher dimension.

In September, I attended summer school "Advanced Scientific Programming in Python", Nikiti, Greece.

In October, I gave an invited talk on conference "Dynamical Systems and Perturbations", dedicated to 70th anniversary of S. Yu. Pilyugin.

4. Teaching

I am currently an Associate Professor at Interdisciplinary Scientific Center J.-V. Poncelet, Moscow but for a bigger part of 2017 I was a postdoc at Centre for Cognition and Decision Making, Higher School of Economics, Moscow. This is an international research unit closely related to the Higher School of Economics's Department of Psychology.

As a member of Neurodynamics Research Group there, in academic year 2016-2017 I supervised thesis project of a 1st year Master student of Higher School of Economics's Department of Psychology, Alexandra Myasnikova. The project was closely related to our work on developing a tool for detection of cross-frequency brain interactions.

In academic year 2017-2018 I am supervising final Master theses of two students, Igor Dubinin from MIPT and Alexandra Myasnikova from HSE. The topics are combinations of mathematics, algorithm improvement and hands-on experiments with electroencephalography data.

In Fall 2016 I taught a few lectures for the course "Computational Neuroscience" on Master's programme "Cognitive Sciences and Technologies: From Neuron to Cognition". I also organized and taught a tutorial seminar "Programming in MATLAB" for bachelors of the Department of Psychology.

This year my direct teaching was a course "Mathematics of brain: mathematical models in neuroscience" at summer school "Contemporary Mathematics 2017", Dubna, Russia, some lectures on summer school "Dynamical Systems", Stola, Slovakia, and a few invited lectures about mathematical models in neuroscience, including the recent one in HSE Nizhny Novgorod.

References

[1] Volk D., Attractors of Piecewise Translation Maps. *arXiv preprint arXiv:1708.03780*, 2017

[2] Volk D., Almost every interval translation map of three intervals is finite type. *Discrete Contin. Dyn. Syst.*, 34(5):2307–2314, 2014.

[3] Nikulin V., Nolte G., and Curio G. Cross-frequency decomposition: A novel technique for studying interactions between neuronal oscillations with different frequencies. *Clinical Neurophysiology*, 123(7):1353–1360, 2012.