Knots and Links in Fluid Flows – from helicity to knot energies

Independent University, Moscow
27-30 April 2015

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY
	27th April	28th April	29th April	30th April
9:30	*Registration*			
9:50	*Opening*			
10:00 - 10:45	Renzo Ricca	Ana Rechtman	Petr Akhmetev	Sofia Lambropoulou
10:50 - 11:35	Dmitry Sokoloff	Yasuhide Fukumoto	William Irvine	Mitch Berger
11:35 - 12:05	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*
12:05 - 12:50	Nechaev Sergei	Daniel Peralta-Salas	Antoine Choffrut	Pierre Dehornoy
12:55 - 13:15	Stathis Anthoniou	Francisco Torres de Lizaur	Chiara Oberti	Egor Illarionov
13:20 - 13:40	Xin Liu	Elena Kudryavtseva	Oleg Karpenkov	Oleg Biryukov
14:00 - 15:30	*Lunch*	*Lunch*	*Lunch*	*Lunch*
15:30 - 16:15	Yoshifumi Kimura	Olga Pochinka	*Free afternoon*	Kenneth Millett (15:40-16:20)
16:20 - 17:05	Gunnar Hornig	Alexey Sossinsky		*Tea break (16:20-16:40)*
17:05 - 17:35	*Tea break*	*Tea break*	*Free afternoon*	Kenneth Millett (16:40-17:20)
17:35 - 17:55	Ana Lecuona	Charles Fougeron		
			Free afternoon	
	Small welcome buffet (18:30)			
	Welcome concert (19:30)	*Social buffet (19:00)*		

Abstracts

Monday 27th April 2015

From helicity to the HOMFLYPT polynomial of fluid knots and links

Opening talk, Monday 10:00-10:45

Renzo Ricca

University of Milano-Bicocca, Italy

In recent years new striking developments on fundamental aspects of topological fluid mechanics have been made. Some of this progress is rooted in a new understanding of the rôle of helicity in fluid flows, and in the development and application of knot theoretical ideas in fluid mechanics. Here, by reviewing fundamental aspects of helicity, we present some of this progress, in the light of the most recent results obtained by the present author and collaborators.

Since Moffatt's original work of 1969, it is well known that helicity (kinetic or magnetic) – an invariant of ideal fluid mechanics – admits topological interpretation in terms of linking numbers (Gauss linking number and Calugareanu-White self linking number). The algebraic and geometric interpretation associated with these numbers makes them useful to detect topological complexity and to relate this complexity to structural changes and energy transfers in fluid flows, even in presence of dissipation. Helicity and energy bounds have been found, and work has progressively diversified, taking various avenues, from work on groundstate energy levels for magnetic knots and links to the study of complex vortex tangles in turbulence [1]. Only recently, though, knot polynomials, such as the Kauffman bracket, the Alexander-Conway and the Jones polynomial, have been derived in classical fluid mechanics. These developments have finally led to the derivation of the HOMFLYPT polynomial as a new powerful invariant of ideal topological fluid mechanics [2].

In presence of dissipation, then, changes of topology take place by the continuous interaction and reconnection of fluid structures. A prototype reconnection process involves an orientation-preserving, anti-parallel recombination of two interacting fluid strands. A mathematical proof that during such a process writhe helicity remains conserved has been given assuming a simple geometric tube model [3], and where there is no exchange of intrinsic twist between the reconnecting strands, even self-helicity is shown to remain conserved. Very recent numerical simulations using the Gross-Pitaevskii equation for a pair of reconnecting quantum vortex rings seem to confirm this new findings, providing exciting grounds for further developments for both classical and quantum systems [4].

References

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Helicity tracers in astrophysical dynamos and labolatory experiments

Monday 10:50-11:35

Dimitry Sokoloff

Moscow State University and IZMIRAN, Russia

Growth rate of large-scale magnetic field in celestial bodies including planets, stars and galaxies is controlled by so-called alpha-effect (e.g. [1, 2]). This process is known as hydromagnetic dynamo. The physical quantity alpha is proportional to correlation time of turbulence or convection and density of hydromagnetic helicity. Integral of this density is well-known in topology as Gauss invariant which gives algebraic sum of weighted linkage of the vortex tubes. Gauss invariant is an inviscid integral of motion as well as topological invariant and a mirror-asymmetric quantity. Due to the alpha-effect, dynamo generated magnetic fields contain a component parallel to the electric current (we remind that usually magnetic field is perpendicular to the electric current). Nonlinear stabilization of hydromagnetic dynamo magnetic field is determined substantially by balance of another physical quantity, i.e. magnetic helicity, which is an inviscid integral of motion, mirror asymmetric quantity and topological invariant (e.g. [3]). Possible role of higher topological invariants is under discussion in current scientific literature. For a quite long time the above topological invariants remain an area of theoretical investigation only however recent efforts of observers (e.g. [4]) and experimentalists (e.g. [5]) open ways to observational and experimental identification of alpha-effect.

References
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q-orhogonal polynomials and knot invariants

Monday 12:05-12:50

Sergei Nechaev
LPTMS, Orsay, France

We consider canonical ensembles of (1+1)-dimensional Dyck paths with "area+length+corners" statistics. Using the combinatorial description, we obtain an explicit expression of corresponding generating function in terms of generalized q-Airy functions. Identifying the generating function with HOMFLY polynomials of torus knots, we are able to study the critical behavior in knot ensembles. We note that reduced "area+length" ensembles are related to the solutions of Burgers equation with small viscosity.

Extending Topological Surgery to Natural Processes

Monday 12:55-13:15

Stathis Antoniou
National Technical University of Athens, Greece

Topological surgery occurs in natural phenomena where two points are selected, forces are applied and the manifold in which they occur changes type. Inspired by such phenomena, we introduce new theoretical concepts which enhance the formal definition of topological surgery with the observed dynamics, thus making the static topological process of surgery an intrinsic and dynamic property of many natural phenomena. More precisely, we introduce local forces in the process of surgery and we define the notions of solid topological surgery (where the interior is filled in). Finally, we embed 2-dimensional topological surgery in the 3-sphere for modeling phenomena which involve more intrinsically the ambient space. We hope that through this study the topology and dynamics of many natural phenomena will be better understood.

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Polynomial knot invariants for fluid knots: from Jones to HOMFLYPT

Monday 13:20-13:40

Xin Liu

Beijing-Dublin International College & Institute of Theoretical Physics, Beijing University of Technology, China

A new method to tackle the topology of fluid knots by using polynomials was derived earlier [1]. There we show that the topology of any vortex tangle can be identified and described by using Jones polynomials of knot theory expressed in terms of helicity. By extending the former interpretation of helicity in terms of linking numbers to the more powerful tool of knot polynomials, we show that this new approach contributes to establish a new paradigm in topological fluid mechanics [2]. Recently this method has been extended to derive also the skein relations of the HOMPLYPT polynomial [3].

Since this is a 2-variable polynomial, the skein relations are expressed in terms of writhe and twist contributions through 2 independent equations. Writhe is associated with addition/subtraction of imaginary local paths, while twist is associated with Dehn's surgery. We discuss some particular examples to show how numerical implementation of the HOMFLYPT polynomial can provide new insight into the dynamics of real fluid flows.

This is joint work with Renzo L. Ricca (U. Milano-Bicocca).

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Reconnection of skewed vortex tubes

Monday 15:30-16:15

Yoshifumi Kimura

Graduate School of Mathematics, Nagoya University, Japan

As an initial stage of vortex reconnection, approach of nearly anti-parallel vortices has often been observed experimentally and studied numerically. Inspired by the recent experiment by Kleckner and Irvine on the dynamics of knotted vortices [1], we have studied the motion of two Burgers vortices situated in a skewed configuration initially which are driven by an axisymmetric linear straining field [2].

In this talk, we first consider the annihilation process of anti-parallel Burgers vortices by extending the Burgers vortex solution, which is a steady exact solution of the Navier-Stokes equation, to a time-dependent case, and then by superposing two such solutions to get an analytic solution. We can demonstrate that during the annihilation process the total vorticity decays exponentially on a time-scale proportional to the inverse of the rate of strain, even as the kinematic viscosity tends to 0. These analytic results are compared with the numerical simulations of two strained vortices with the vortex-vortex nonlinear interaction by Buntine and Pullin [3].

Then we proceed to the skewed case. Due to the straining action of velocity, two Burgers-type vortices originally placed symmetrically in a skewed configuration get closer to a single line being parallel each other while the crossing parts tend to annihilate. Using an analytic solution, we can monitor this process graphically. Helicity is calculated also analytically during this process, which decays to zero exponentially as reconnection occurs.

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Relaxation of Braided Magnetic Fields

Monday 16:20-17:05

Gunar Hornig
University of Dundee, United Kingdom

Magnetic fields in the solar atmosphere are often twisted and braided due to the turbulent motion of the plasma on the Sun's surface. These "magnetic braids" typically evolve towards a minimum energy state, a so-called force-free state. This relaxation process can be of turbulent nature and involve magnetic reconnection processes, which change the topology of the magnetic field. The relaxation is constrained by certain topological invariants, which survive the turbulent reconnection processes. Until recently it was believed that magnetic helicity is the only invariant which has this property. We present results of numerical simulations on the relaxation of braided magnetic fields in plasmas of high magnetic Reynolds number, which show that there are further constraints beyond the conservation of the total magnetic helicity. A generalised flux function can be defined for magnetic braids [1], which helps to understand qualitatively and quantitatively the relaxation process and can be used to predict possible relaxed states.

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The colored signature of a splice link

Monday 17:35-17:55

Ana Lecuona
Institut de Mathmatiques de Marseille, University of Marseille, France

The splice of two links is an operation defined by Eisenbund and Neumann which generalizes several other operations on links, such as the connected sum, the cabling or the disjoint union. The precise definition will be given in the talk but the rough idea goes as follows: the splice of the links L' and L'' along the components K' and K'' is the link $(L' \setminus K') \cup (L'' \setminus K'')$ obtained by identifying the exterior of K' with the exterior of K''. There has been much interest to understand the behavior of different link invariants under the splice operation (genus, fiberability, Conway polynomial, Heegaard-Floer homology among others) and the goal of this talk is to present a formula relating the colored signature of the splice of two oriented links to the colored signatures of its two constituent links. This is a joint work with Alex Degtyarev and Vincent Florens.

Tuesday 28th April 2015

The dynamics of the minimal set of Kuperberg's plug

Tuesday 10:00-10:45

Ana Rechtman

IRMA, University of Strasbourg, France

In 1993 K. Kuperberg constructed examples of C^{∞} and real analytic flows without periodic orbits on any closed 3-manifold. These examples continue to be the only known examples with such properties. The construction is based on the construction of a plug. A plug is a manifold with boundary of the type $D^2 \times [0,1]$ endowed with a flow that enters through $D^2 \times \{0\}$, exits through $D^2 \times \{1\}$ and is parallel to the rest of the boundary. Moreover, it has the particularity that there are orbits that enter the plug and never exit, that is there are trapped orbits. The closure of a trapped orbit limits to a compact invariant set contained entirely within the interior of the plug. This compact invariant set contains a minimal.

The first construction of a plug without periodic orbits was done by P. Schweitzer. This plug is constructed from the minimal set that is the Denjoy flow on the torus, implying that the flow of the plug is only of class C^1. Kuperberg's construction is completely different in nature. I will present a study of the minimal set, its dynamics and topology. This is a joint work with Steve Hurder (University of Illinois at Chicago).

Are all the topological invariants representable as cross-helicities?

Tuesday 10:50-11:35

Yasuhide Fukumoto

Institute of Mathematics for Industry, Kyushu University, Japan

Integrals of an arbitrary function of the vorticity, two-dimensional topological invariants of an ideal barotropic fluid, take different guise from the helicity. Noether's theorem associated with the particle relabeling symmetry group leads us to a unified view that all the topological invariants of a barotropic fluid are variants of the cross helicity. Baroclinic fluid flows admit, as the Casimir invariants, a class of integrals including an arbitrary function of the entropy and the potential vorticity. These Casimir invariants are also viewed as the cross helicity. We show that the similar holds for the MHD.

Existence of knots and links in stationary fluid flows

Thursday 12:05-12:50

Daniel Peralta-Salas

ICMAT (Institute of Mathematical Sciences), Universidad Autónoma de Madrid, Spain

The motion of particles in an ideal fluid is described by its velocity field $u(x,t)$, which satisfies the Euler equations

$$\partial_t u + (u \cdot \nabla) u = -\nabla p\,, \qquad \mathrm{div} u = 0$$

for some pressure function $p(x,t)$, which is also an unknown of the problem. A solution u to the Euler equations is called *stationary* when it does not depend on time. As is well known, a vector field $u(x)$ is a stationary solution of the Euler equations if and only if it satisfies the system of PDEs:

$$u \times \omega = \nabla B\,, \qquad \mathrm{div}\, u = 0\,,$$

where

$$B := P + \frac{1}{2}|u|^2$$

is the Bernoulli function and $\omega := \mathrm{curl}\, u$ is the vorticity. In this talk we will be concerned with the existence of knotted structures in stationary solutions of the Euler equations in $\mathbb{R}^3$.

More precisely, let us recall that the integral curves of the vorticity are called *vortex lines*, and a solid torus in $\mathbb{R}^3$ that is the union of vortex lines is a *vortex tube*. The analysis of knotted and linked vortex lines and vortex tubes for solutions to the Euler equations has attracted considerable attention since the foundational works of Helmholtz and Kelvin in the 19th century. In fact these structures have been recently realized experimentally by Kleckner and Irvine at Chicago [4].

The aim of this talk is to outline some recent results showing the existence of stationary solutions of the Euler equations with a prescribed set of knotted and linked vortex lines and vortex tubes. A key feature of these results is that they do not consider arbitrary solutions to the stationary Euler equations, but a particular class known as Beltrami fields. These are the solutions of the equation

$$\operatorname{curl} u = \lambda u$$

in $\mathbb{R}^3$ for some nonzero constant λ. This immediately implies that u is an analytic, divergence-free vector field. Notice that the celebrated Arnold's theorem on the structure of the stationary solutions to the Euler equations [1] does not apply to Beltrami fields because their associated Bernoulli function is identically constant, so in particular Beltrami fields do not need to be integrable.

Theorem 1 [2] Let L be a locally finite link in $\mathbb{R}^3$. Then for any nonzero real constant λ one can deform L with a diffeomorphism Φ of $\mathbb{R}^3$, arbitrarily close to the identity in the C^k norm, so that $\Phi(L)$ is a set of closed vortex lines of a Beltrami field u, which satisfies $\operatorname{curl} u = \lambda u$ in $\mathbb{R}^3$.

Theorem 2 [3] Let T be a finite collection of (pairwise disjoint, but possibly knotted and linked) tubes in $\mathbb{R}^3$. Then there is a diffeomorphism Φ of $\mathbb{R}^3$ so that $\Phi(T)$ is a set of vortex tubes of a Beltrami field u, which satisfies $\operatorname{curl} u = \lambda u$ in $\mathbb{R}^3$, and falls off at infinity as $|x|^{-1}$.

A few remarks on these results are in order. First, the link L in Theorem 1 is allowed to have infinitely many components, while we need to demand that the set T in Theorem 2 consists of finitely many pieces. Second, the diffeomorphism Φ of the second theorem is not generally close to the identity, in fact each vortex tube in $\Phi(T)$ is *thin* in the sense that its width is much smaller than its length. Third, the behavior of the Beltrami field u at infinity in Theorem 1 is not controlled, unless we assume that the link L is finite, in which case u falls off at infinity as $1/|x|$.

The proofs of these results combine ideas from partial differential equations and dynamical systems. In particular, they require the extension of some classical tools to this context: a Cauchy-Kowalewsky theorem for the curl operator, a KAM theorem for Beltrami fields and a Runge-type global approximation theorem with controlled behavior at infinity.

References
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Knotted structures in high-energy Beltrami fields on the torus and the sphere

Tuesday 12:55-13:15

Francisco Javier Torres de Lizaur

ICMAT(Institute of Mathematical Sciences), Universidad Autónoma de Madrid, Spain

A stationary ideal fluid flow on a three dimensional Riemannian manifold (M, g) is described by a velocity field $u(x)$ which satisfies the stationary Euler equations

$$i_\omega i_u \mu = dB\,, \qquad d i_u \mu = 0$$

where μ is the volume form on (M, g), $B(x)$ is the so-called Bernoulli function and $\omega := \mathrm{curl} u$ is the *vorticity field*.

A major topic in topological hydrodynamics, going back to Lord Kelvin [5], concerns the existence of knotted and linked structures in stationary fluid flows, specially of knotted and linked stream and vortex lines (integral curves of the velocity and vorticity fields, respectively) and vortex tubes (bounded domains whose boundaries are invariant tori of the vorticity field).

In light of his structure theorem for the stationary Euler equations [1], V. Arnold suggested that this rich topological behaviour was to be sought within a particular class of solutions called *Beltrami fields*.

A Beltrami field is an eigenfield of the curl operator, $\mathrm{curl} u = \lambda u$, or equivalently, a stationary flow whose velocity and vorticity are proportional. If they are everywhere non zero, they can also be seen as the Reeb field of a contact form.

A. Enciso and D. Peralta-Salas showed [2,3] that knots and links of any type arise as vortex lines and vortex tubes of Beltrami fields in $\mathbb{R}^3$. Their techniques, relying on a Runge-type approximation theorem, cannot be applied to compact manifolds. In this talk we will sketch a proof of the fact that on the round sphere $\mathbb{S}^3$ and on the flat torus $\mathbb{T}^3$ high-energy Beltrami fields can indeed exhibit arbitrarily knotted and linked vortex lines and tubes. More precisely:

Theorem Let $\mathcal{S}$ be a finite union of (pairwise disjoint, but possibly knotted and linked) closed curves and tubes in $\mathbb{S}^3$ or $\mathbb{T}^3$. In the case of the torus, we assume that $\mathcal{S}$ is contained in a contractible subset of $\mathbb{T}^3$. Then for infinitely many large enough eigenvalues λ of the curl operator there exists a Beltrami field u with energy λ and a diffeomorphism Φ of $\mathbb{S}^3$ or $\mathbb{T}^3$ such that $\Phi(\mathcal{S})$ is a union of vortex lines and vortex tubes of u. Furthermore, this set is structurally stable.

One key to the proof is the growing degeneracy of the spectra of the curl operator on the round sphere and on the flat torus: our strategy does not work in general Riemannian 3-manifolds. Another feature of the result is that it gives a very precise understanding of the nature of the diffeomorphism Φ: its effect is to rescale a contractible subset containing $\mathcal{S}$ to have diameter of order λ^{-1}.

This is a joint work with Alberto Enciso and Daniel Peralta-Salas [4].

References

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Topological invariants of ideal magnetic fields are functions in helicity or have no derivative with C^1-continuous density

Tuesday 13:20-13:40

Elena Kudryavtseva

Moscow State University, Russia

By an ideal magnetic field in a magnetic tube we mean a closed 2-form B in the full torus $V = D^2 \times S^1$ whose restriction to the boundary torus $\partial V = S^1 \times S^1$ vanishes, while the restriction to any fibre $D^2 \times \{*\}$ has no zeros, moreover the integral of B over this fibre is 1. By a topological invariant of magnetic fields in V we mean a real-valued function $I = I(B)$ on the space of such 2-forms such that $I(B) = I(h^*B)$ for any diffeomorphism $h : V \to V$ isotopic to the identity. An important example of topological invariants is helicity, which is the averaged linking number of magnetic lines. By using [1], we prove that a topological invariant $I = I(B)$ is either a function in helicity or has no derivative with C^1-continuous density [2]. Similar properties of the helicity and the closely related Calabi invariant were proved in [3], [4].

References
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On heteroclinic separators of magnetic fields in electrically conducting fluids

Tuesday 15:30-16:15

Olga Pochinka, with Viatcheslav Grines, Evgenii Zhuzhoma
National Research University Higher School of Economics, Nizhny Novgorod, Russia

The paper is about the topological structure of magnetic fields in electrically conducting fluids. The interactions between a magnetic field and a given conducting fluid are adequately described by the equations of magnetohydrodynamics (MHD). The evolution of a magnetic field $\vec{\boldsymbol{H}}$ in a moving conductor is described by the induction equation

$$\frac{\partial \vec{\boldsymbol{H}}}{\partial t} = rot\ \left[\vec{\boldsymbol{v}}\vec{\boldsymbol{H}}\right] + \eta \nabla^2 \vec{\boldsymbol{H}}$$

where $\vec{\boldsymbol{v}}$ is the velocity of conducting fluid (match smaller than the speed of light c) and η is the magnetic diffusivity of the medium. Literature on MHD often uses the magnetic induction $\vec{\boldsymbol{B}}$, for which $\vec{\boldsymbol{B}} = \mu\vec{\boldsymbol{H}}$ where μ is the magnetic permeability of the fluid (it is proportionally to $\frac{1}{\eta}$).

Hannes Alfvén [1] showed that in a fluid with a large magnetic Reynolds number (it is proportionally to μ) the field lines move as though they are "frozen" into the medium. As a consequence the topological structure of the magnetic field preserves under a short-time steady motion. The concept of frozenness implies the existence of locations where the magnetic field changes in either directions or magnitude or both. As a consequence, null points (i.e. points where the magnetic field vanishes) can occur in the magnetize medium. The null points in three dimensions typically look like a saddle of the vector field $\vec{\boldsymbol{H}}$. Moreover, such a saddle is a conservative one with nonzero eigenvalues λ_1, λ_2, λ_3 that satisfy the condition $\lambda_1 + \lambda_2 + \lambda_3 = 0$, due to $\nabla \cdot \vec{\boldsymbol{H}} = 0$. Two quite distinct families of field lines pass through a null point: the null *spine* is the isolated field line and the null *fan* is a surface. A magnetic line joining two nulls and representing the intersection of two fans is called a *separator* [3], [4]. A separator is *heteroclinic* if it is a transversal intersection of the fans.

The topological structure of a magnetic field is defined by null points, spines, fans, and separators, the union of those forming the so-called *skeleton* of the magnetic field. To study the global magnetic topology we have to consider first the problem of the existence of null points and separators. We suggest to approach this problem in the following way. We consider a body in plasma (i.e. a part of plasma) of special kind and we study movements of plasma such that all the boundary components of the body move inside or outside of the body so that after some time interval all the boundary components are parallel to the initial boundary components. The statement of the problem implies that during the movement the topological structure of the magnetic field remains unchanged, therefore it is natural to assume the skeleton of the magnetic field to be invariant with respect to the movement of plasma. Notice that we do not demand all the points of the skeleton to be fixed, we just assume that the points on the skeleton, while moving, remain on the skeleton. We also assume that the null points are not only null hyperbolic points of the magnetic field but they also are null hyperbolic points of the movement of plasma. Taking into consideration that the Kupka-Smale theorem from Dynamical System Theory states that for every typical movement all periodical points (including fixed points) are hyperbolic, we consider only the class of typical movements of plasma.

The suggested approach lets us apply methods and results of Dynamical Systems Theory because it turns out to be possible to extend the motion of plasma to some 3-manifold in such a way that one gets a classical dynamical system. Though doing so we loose information on the structure of the magnetic field outside the considered body, on the other hand we get an instrument to study its structure in some part of space. Besides, the global structure on the considered manifold sheds light on possible real structures of the magnetic field. The suggested approach to the problem and the method lead to theoretical issues of Dynamical Systems Theory whose solution is of interest in its own.

Let M^2_p be an orientable closed surface of genus $p \geq 0$ smoothly embedded in $\mathbb{R}^3$. Two surfaces $M^2_{p_1,1}$, $M^2_{p_2,2} \subset \mathbb{R}^3$ are called *parallel* if they bound the set which is homeomorphic to $M^2_p \times (0;1)$. As a consequence, $p_1 = p_2 = p$ and $M^2_{p,1} \cap M^2_{p,2} = \emptyset$. The closure of $M^2_p \times [0;1]$, in short a *fat surface*, is a 3-manifold with the boundary $M^2_{p,1} \cup M^2_{p,2}$. In particular, $\mathcal{S} = \mathbb{S}^2 \times [0;1]$ is a *fat sphere* where $\mathbb{S}^2$ is the 2-sphere. One of its boundary 2-spheres, say $S^2 \times \{0\} = S_{int}$, bounds a open 3-ball in $\mathbb{R}^3$ which is disjoint from $\mathcal{S}$. We call S_{int} *interior* while the sphere $S^2 \times \{1\} = S_{ext}$ is called *exterior.* Let B_{int} and B_{ext} be the 3-balls bounded by S_{int} and S_{ext} respectively.

Let G_p be a fat surface bounded by parallel surfaces $M^2_{p,1}$ and $M^2_{p,2}$ and let it contain two 2-spheres S_1 and S_2 such that $S_1 \cap S_2 = \emptyset$ and the balls B_1 and B_2, bounded by S_1 and S_2 respectively, do not intersect. Denote by $\mathcal{M}$ the fat surface G_p with two holes, i.e. the set $G_p \setminus \{B_1 \cup B_2\}$.

Let M be either $\mathcal{S}$ or $\mathcal{M}$ which is smoothly embedded into $\mathbb{R}^3$. Suppose M to be a part of space with plasma of some astrophysical object with a magnetic field $\vec{\boldsymbol{B}}$. Denote by $\vec{\boldsymbol{B}}_0$ the restriction of $\vec{\boldsymbol{B}}$ to M and let all the null points of $\vec{\boldsymbol{B}}$ be typical (i.e. hyperbolic). As the consequence we have that M has only finitely many null points.

We assume that

1. the separatrices of the null points intersect transversally (if at all);
2. if $M = \mathcal{M}$ the separatrices intersect the components S_1 and S_2 transversally (if intersect at all).

A map $f_0 : M \to f_0(M) \subset \mathbb{R}^3$ is said to be an *frozen-motion* if it satisfies the following conditions:

(a) f_0 is an orientation preserving diffeomorphism to its image. The non-wandering set of f_0 consists of fixed hyperbolic points which coincide with the null points of the vector field $\vec{\boldsymbol{B}}_0$.

(b) The boundary components of $f_0(M)$ are pairwise disjoint from the boundary components of M.

(c) If $M = \mathcal{S}$ then the interior boundary sphere maps inside $\mathcal{S}$ while the external boundary sphere maps outside $\mathcal{S}$, i.e. $f_0(S_{int}) \subset \mathcal{S}$ and $f_0(S_{ext}) \subset \mathbb{R}^3 \setminus (\mathcal{S} \cup B_{int})$. If M is the fat surface with holes $\mathcal{M}$ then one of the boundary spheres, say S_1, maps inside $\mathcal{M}$ while the other boundary sphere S_2 maps outside $\mathcal{M}$ and one of the boundary surfaces $M^2_{p,i}$, say $M^2_{p,1}$, maps inside $\mathcal{M}$ while the other boundary surface $M^2_{p,1}$ maps outside $\mathcal{M}$. Moreover, the restriction $f_0|_{M^2_{p,i}} : M^2_{p,i} \to f_0(M^2_{p,i})$ is homotopy trivial for each $i = 1, 2$.

(d) The fixed points of f_0 are of the same type as the null points of the field $\vec{\boldsymbol{B}}_0$ and the fans and the spines are invariant with respect to f_0.

Our main result for frozen-motion is the following.

Theorem [2] Let $f_0 : M \to f_0(M) \subset \mathbb{R}^3$ be an frozen-motion of a fat surface M which belongs to some domain of plasma with a magnetic field $\vec{\boldsymbol{B}}_0$. Then the fan of each null point has in M at least one heteroclinic separator.

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Normal forms of wire knots and the Euler functional: a new approach to knot energy

Tuesday 16:20-17:05

Alexey Sossinsky

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The idea of considering the energy of knots arose in fluid mechanics in two seminal papers of Keith Moffatt, was further developed in Japan, especially by Fukuhara, Jun O'Hara [1], who defined what is now known as Moebius energy, a functional on knots studied further by Michael Freedman et al [2], [3]. We will briefly recall the O'Hara functionaland related results, and then describe our approach to knot energy.

Simple physical experiments show that a deformed knotted resilient elastic wire always returns to its equilibrium shape (which we call the normal form of the wire knot, [4]). The aim of our research (joint work with S. Avvakumov and O. Karpenkov [5], [6], [7]) is to construct a mathematical model of such physical knots. This is done by supplying the space of knots with an energy functional and performing gradient descent w.r.t. the functional. The energy functional that we use is the sum of the Euler functional (the integral of the square of the curvature along the curve defining the knot) and a simple repulsive functional (that forbids crossing changes). Using a discretized version of gradient descent along that functional, we construct an algorithm that brings the knot to the shape that minimizes its energy. The algorithm is implemented in a computer animation that shows the isotopy bringing the knot to normal form.

We will show some of these animations and demonstrate a few of the physical experiments with wire knots, compare the results of physical and mathematical experiments, outline how our algorithm can be used in practice to recognize knots and to unravel trivial knots, and assess the advantages and disadvantages of our approach as compared with that of O'Hara.

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Cries and whispers in the wind-tree forest

Tuesday 17:35-17:55

Charles Fougeron

Université Paris 7, France

In [3] P. and T. Ehrenfest introduced a model of the wind blowing in a periodic forest. Suppose we have square shaped trees positioned on every integer points of the plane, we now get interested in the bouncing of a given particule of the wind. We assume it interacts only with the trees and not with other particles. When hitting a tree, it will bounce according to the laws of reflection. How will its trajectory behave asymptotically?

Recently advances have been made in understanding the behaviour of this flow. Namely, V. Delecroix P. Hubert and S. Lelivre have computed in [1] the speed at which it goes far away from its starting point, its diffusion rate. In this case, it is 2/3 ! The starting point of their computation is to process with the classical development of the surface. As we would do for billard on a square, we strengthen up the flow by introducing surfaces symmetric to the edges of the billard. This creates a translation surfaces on which we have plenty of tools. In particular the diffusion rate is a Lyapunov exponents of this surface.

A natural question asked by Yoccoz, was to know if we can find shapes of trees for which the diffusion rate goes to zero or to one. Few months ago, A. Zorich and V. Delecroix answered almost completly to this question in [2], finding a nice shape for which it conjecturally goes to one, and showing that if we take a circle approximated by squares of size going to zero, the diffusion rate will go to zero. This gives a flavour of how the theory can be surprising, and hard to handle with intuition, since when we look at the diffusion rate for circles obstacles, the diffusion rate is $1/2$.

These results use extensively the latest advances in understanding the $SL_2\mathbb{R}$ invariant subspaces of Teichmülcer spaces, in particular the work of Eskin-Mirzakhani [5] and their Lyapunov exponents which can are better apprehended since the work of Eskin-Kontsevich-Zorich [4].

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Wednesday 29th April 2015

Higher Helicity of Magnetic Lines and Arf-invariants

Wednesday 10:00-10:45

Petr Akhmet'ev

Pushkov Institure of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation of the Russian Academy of Sciences, Moscow, Russia

V. I. Arnold in 1984 formulated the following problem: "To transform the asymptotic ergodic definition of the Hopf invariant of divergence-free vector fields to the theory of S. P. Novikov which generalizes the Whitehead product of homotopy groups of spheres"'. Algebraic commutators, which are used to define the higher invariants of classical links, are particular Whitehead products in homotopy groups of spheres.

We present an additional motivation of the Arnold Problem, which is based on mean magnetic field theory. We use geometrical considerations due to K. Moffatt and formulate properties of invariants in ideal MHD, which are asymptotic and ergodic properties.

We recall the definition of the quadratic helicity invariant and of the higher asymptotic ergodic M-invariant. We present a simpler new proof (in part) that the M-invariant is ergodic. The M-invariant is a higher invariant, this means that for the magnetic field with closed magnetic lines the invariant is not a function of pairwise linking numbers of the magnetic lines. This property is based of the following fact: an arithmetic residue of the M-invariant for a triple of closed magnetic lines, which is a model of a link with even pairwise linking numbers, coincides with the Arf-invariant (the definition of the Arf-invariant will be recalled).

The new results concern magnetic fields on closed 3-dimensional manifolds and use the M-invariant. The manifolds with magnetic field that we consider are not, generally speaking, simply-connected. This manifold is assumed homogeneous and is a rational Poincaré sphere. One can try to transform results on the asymptotics and ergodicity of the M-invariant for the magnetic fields on the standard sphere S^3 to an arbitrary rational homology sphere Σ. To make this idea precise we generalize the Arf-invariants of classical semi-boundary links (including the Arf-Brown $\mathbb{Z}/8$-invariant) and we introduce a new Arf-invariant, called the hyperquaternionic Arf-invariant.

These results show relationship between the M-invariant and homotopy groups of spheres. It is well-known that the helicity invariant is a specification of the Hopf invariant. The Hopf invariant determines the homotopy group $\pi_3(S^2)$, the stabilization of this homotopy group is denoted by Π_1. The group Π_1 contains the only non-trivial element with the Hopf invariant one.

The Arf-invariant describes the stable homotopy group Π_2 via the geometrical approach due to L. S. Pontrjagin. The Arf-Brown invariant describes the 2-torsion of the stable homotopy group Π_3, this result follows from V. A. Rokhlin's theorems. The hyperquaternionic Arf-invariant describes the 2-torsion of the stable homotopy group Π_7. This group was calculated by J.-P. Serre using the algebraic approach. The complexity of the M-invariant relates with the fundamental group of rational homology sphere Σ.

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The life of a vortex knot: Linking coiling and twisting across scales

Wednesday 10:50-11:35

William Irvine

University of Chicago, USA

Can you take a vortex loop-akin to a smoke ring in air - and tie it into a knot or a link? The possibility of such knottiness in a fluid has fascinated physicists and mathematicians ever since Kelvins vortex atom hypothesis, in which the atoms of the periodic table were hypothesized to correspond to closed vortex loops of different knot types. More recently, the knottiness (Helicity) of a fluid has re-emerged as a conserved quantity in many idealized situations (such as Euler fluids and ideal plasmas) offering the potential for new fundamental insights. In the real physical counterparts to these systems, progress has however been hindered by lack of accessible experimental systems. I will tell of how to make a vortex knot and link in water, in the wave function of a superfluid (on a computer) and of what happens thence. In particular, I will talk about how helicity conservation plays out by exchanging linking, coiling and twisting across scales.

Geometry in the Euler equations of hydrodynamics: h-principle and convex integration

Wednesday 12:05-12:50

Antoine Choffrut

University of Edinburgh, United Kingdom

The following dichotomy between rigidity and flexibility is now well known in geometry: while uniqueness holds for smooth solutions to the isometric embedding problem, the set of solutions becomes unimaginably large if one allows rough ones. What is surprising is that this dichotomy holds for problems coming from mathematical physics, and in particular the Euler equations of fluid dynamics.

In this (mainly expository) talk I will explain the h-principle and the method of convex integration. Convex geometry is the heart of the matter and profuse figures will attempt to illustrate the difficulties and how to tame them.

Energy and helicity of magnetic torus knots and unknots

Wednesday 12:55-13:15

Chiara Oberti

University of Milano-Bicocca, Italy

We consider steady magnetic fields in the shape of torus knots and unknots. Torus knots are closed, space curves, that are wrapped uniformly around a mathematical torus, p times in the toroidal (longitudinal) direction and q times in the poloidal (meridian) direction, with $1 < p < q$, p, q co-prime integers [1]. The winding number $w = q/p$ is taken as a measure of knot complexity. Three different approaches to calculate the helicity associated with a magnetic torus knot in ideal MHD are compared to show that the helicity of toroidal knots/unknots is dominated by writhe contribution. By measuring helicity in terms of the Calugareanu-White linking number [2], we show that since the transition of a torus knot/unknot through an inflectional state produces a jump in the sum of writhe and normalized total torsion (the so-called Pohl's self-linking number) [3], helicity conservation is guaranteed by an equal and opposite jump in the intrinsic twist given by the number of poloidal coils [4]. We find an analytic lower bound for the magnetic energy, in terms of helicity and a quantity that depends only on the geometry of the torus knot/unknot. The dependence of geometric energies, associated with total squared curvature and torsion, on the winding number is also analysed.

This work is part of a PhD project carried out under the supervision of R.L. Ricca.

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Energies of knots and graphs

Wednesday 13:40-14:00

Oleg Karpenkov

University of Liverpool, United Kingdom

Energies of knots (graphs) are functionals on the space of smooth or piecewise smooth knots (graphs). They were introduced in order to simplify embeddings of knots using gradient flows corresponding to these functionals. In the talk we will study several energy functionals and their minimizers.

Thursday 30th April 2015

The invariants for classical knots and links from the Yokonuma-Hecke algebras

Thursday 10:00-10:45

Sofia Lambropoulou

National Technical University of Athens, Greece

The Yokonuma-Hecke algebras, $Y_{d,n}(q)$, are quotients of the framed braid group and they include the Iwahori-Hecke algebra, $H_n(q)$, for $d = 1$. In the talk we will introduce the algebras $Y_{d,n}(q)$ and the Juyumaya traces tr_d defined on them. From the traces tr_d and for every $d \in N$ we derive invariants for knots and links upon imposing a condition on the trace parameters. The question is how these invariants compare with the 2-variable Jones or Homflypt polynomial. We will show that for knots they are topologically equivalent to the Homflypt polynomial. However, they do not seem to demonstrate the same behavior on links.

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Self-Organized Braiding in Solar Coronal Loops

Thursday 10:50-11:35

Mitch Berger

Newcastle University, United Kingdom

Observations of the solar atmosphere suggest the presence of magnetised clouds of plasma entrained along magnetic field lines. These clouds take the form of loops with fine scale internal structure. We assume that they consist of several internal strands which twist and braid about each other. Reconnection between the strands leads to small flares and heating of the loop to x-ray temperatures. Using a method of generating and releasing braid structure similar to a forest fire model, we show that the reconnected field lines evolve to a self-organised critical state [1]. In this state, the frequency distributions of coherent braid sequences as well as flare energies follow power law distributions. We demonstrate how the presence of net helicity in the loop alters the distribution laws.

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Left-handed flows

Thursday 12:05-12:50

Pierre Dehornoy

Institut Fourier, Université de Grenoble-Alpes, France

Linking number is the oldest invariant of 3-dimensional links. As pointed out by Moffatt [6] and Arnold [1], helicity is a natural extension of the linking number to 3-dimensional flows. Namely it measures a mean value of the linking numbers of all orbits of a given flow. For example, the negativity of the helicity says that most pairs of orbits are negatively linked.

Left-handedness [4] is a property of certain 3-dimensional flows, which says that every pair of periodic orbits is negatively linked. It is a very strong property that many (most?) flows do not have. On the other hand, if a flow is left-handed, it has many interesting properties, in particular every finite collection of periodic orbit bounds a so-called Birkhoff section (a global section for the flow in the complement of the boundary of the section). In other words a left-handed flow can be reduced to the suspension of a diffeomorphism of a surface in as many ways as one can hope.

Despite the strong constraints in the definition, left-handed flows do exist, as for example the Lorenz vector field [2,5], the Hopf vector field, or several geodesic flows [3]. The goal of the talk is to give a rough introduction to this notion.

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Partitioning of magnetic field into flux tubes

Thursday 12:55-13:15

Egor Illiaronov (with D. Sokoloff, P. Akhmetiev, M. Georgoulis, A. Smirnov)

Moscow State University, Russia

Application of some topological concepts such as twist, writhe, linkage and helicity to astrophysical magnetic fields implies a possibility to trace magnetic field lines and magnetic flux tubes. The point is that observational basis is very limited and typically we observe only specific components of magnetic field. Extrapolation enables to reconstruct magnetic field in given 3D volume and topological structure of the field obtained contains important information about generation processes. We discuss several problems that arise on the way of practical computation of topological invariants and consider different approaches.

An explicit formula for 3-braid entropy

Thursday 17:35-17:55

Oleg Biryukov

Kolomna State Pedagogical University, Russia

Braids on 3 strands generate a group which acts on the standard 2-disc with 3 small discs removed. The entropy of this action is calculated as the trace of a representation matrix in the group SL(2,Z). An explicite formula for this trace and for the entropy of the action is presented. Properties of polynomials is the formula are investigated.

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Knotting and Linking in Fluid Flows

Globus seminar, Thursday 15:40-17:20

Kenneth Millett

University of California, Santa Barbara, CA, USA

Vortex lines in a fluid flow are modeled by systems of mathematical curves that are entangled through their knotting or linking, both local and global. Their mutual interference is implicated in large-scale effects making their characterization and quantification an objective of substantial interest. In this study, two mathematical streams are brought together: First, the knotting of open chains used to characterize knotting in, for example, proteins [1][2][3]. Second, the periodic linking measures used to study entanglement in periodic boundary condition (PBC) models of polymer melts [4][5].

The first derives from the application of knot polynomial invariants [6]. We will describe the fundamental features of knotting and the extension of these ideas that has lead us to our study of knotting in open filament structures such as proteins or fluid flow lines. The second is inspired by the Gauss linking number and is related to the helicity [7]. We will describe the extension of these to the context of periodic boundary condition models and the creationg, by Panagiotou, of the periodic linking number. Its analysis provides the foundation upon one can determine an associated linking matrix and calculate measures of entanglement as a function of the scale of the vortex filaments and the consequences of confinement or global forces.

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