

**International Conference**  
***”Flows on surfaces, symbolic dynamics and dynamics in  
moduli spaces.”***

Laboratoire CNRS J.-V.Poncelet, Moscow  
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**ABSTRACTS**

**On Rauzy gasket**

Pierre Arnoux

Institut de Mathématiques de Luminy, Marseille, France

Strict episturmian sequences (sequences of complexity  $2n+1$  on 3 letters, with exactly one left-special word and right-special word of any length) are a natural generalization of rotation sequences; they are uniquely ergodic, and the vector of frequencies of the 3 letters play the same role as the rotation number for rotation sequences.

The Rauzy gasket is the closure of this set of frequency vectors; it is a compact subset of the standard simplex. In this talk, we will study some properties of the Rauzy gasket, its relation to the Sierpinski triangle and the Apollonian gasket, and its relation to some 2-dimensional continued fraction algorithm; in particular, we will prove that it has measure zero, and it is a solution of an Iterated Function System, and a repeller for an expanding map of the simplex.

**Symbolic dynamics of a traffic map**

Michael Blank

Institute for Information Transmission Problems, Moscow, Russia

We study ergodic properties of a family of traffic maps acting in the space of bi-infinite sequences. The corresponding dynamics mimics the motion of vehicles in a simple traffic flow, which explains the name. Using connections to topological Markov chains we obtain nontrivial invariant measures and calculate the topological entropy. Technically these results are related to the construction of measures of maximal entropy via measures uniformly distributed on periodic points of a given period. In distinction to conventional results the limiting measure in our case is non-ergodic.

**Connected components of the strata of the moduli space of meromorphic differentials.**

Corentin Boissy

Université Paul Cézanne Aix-Marseille III, France

A holomorphic one-form on a Riemann surface defines a flat metric with conical singularities. Such kind of metrics, and the corresponding moduli spaces, have been widely studied in the last 30 years

In this talk, I will investigate flat structures on Riemann surfaces defined by meromorphic one-forms. I will give a description of the connected components of the corresponding moduli spaces.

**Interval exchange extending substitutive dynamical systems**

Xavier Bressaud

Université Paul Sabatier, Toulouse, France

This is a joint work with Yann Julian.

Let  $\sigma$  be an invertible and primitive substitution with maximal index and consider the corresponding substitutive dynamical system, i.e. shift on a  $\sigma$ -invariant subshift  $\Sigma$ . I will show that there exists an Interval Exchange Transformation on the circle whose coding factorizes onto  $\Sigma$ .

This result generalizes a relationship between the Tribonacci substitutive system (Rauzy fractal) and the Arnoux-Yoccoz interval exchange transformation explored by Pierre Arnoux and illustrated by the construction of a Peano curve in the Rauzy fractal.

**Diffusion in some periodic billiards**

Vincent Delecroix

Université Paris-7, France

The windtree model is a periodic billiard in the plane where at each integer point lies a rectangular scatterer. We prove that the maximum distance reached by the ball until time  $T$  has roughly the size of  $T^{2/3}$  where the exponent  $2/3$  is a Lyapunov exponent of the Kontsevich-Zorich cocycle. We also investigate the computation of this exponent and see how it can be generalized to other periodic billiards.

## **On combinatorial properties of Rauzy graphs: trees constructed of cycles**

Mikhail Dubashinsky

Chebyshev Laboratory, Saint-Petersburg, Russia

Consider Rauzy induction mappings  $a$  and  $b$  acting on the set  $S_m^0$  of all irreducible permutations of numbers  $1, \dots, m$ . For  $\pi \in S_m^0$  denote by  $R(\pi)$  the set of all permutations that we can obtain from  $\pi$  by applying operations  $a$  and  $b$ . The set  $R(\pi)$  has a natural structure of oriented graph according to the action of operations  $a$  and  $b$  on this set. Our main result is the following. Graph  $R(\pi)$  is "a tree constructed of cycles" if and only if the permutation  $\pi_0: i \mapsto m + 1 - i$  ( $i = 1, \dots, m$ ) belongs to  $R(\pi)$  (in this case  $R(\pi) = R(\pi_0)$ ).

## **On interrelation between dynamics and topology of ambient manifold**

Viacheslav Grines

Nizhny Novgorod State Agricultural Academy, Russia

We establish some relations between topology of a 3-manifold  $M^3$  with the dynamics of Morse-Smale diffeomorphism acting on it. These relations are connected with the number  $g_f = \frac{r_f - l_f + 2}{2}$ , where  $r_f$  is the number of saddle periodic points and  $l_f$  is the number of sink and source periodic points of  $f$ . We give a topological classification of closed orientable three-manifolds admitting Morse-Smale flows without heteroclinic curves. These manifolds are either three-dimensional sphere if  $g_f = 0$ , or the connected sum of  $g_f$  copies of  $S^2 \times S^1$ . Another relation between the number of  $g_f$  and the topology of  $M^3$  is found in the case when the diffeomorphism  $f$  is gradient-like and has tame one-dimensional bundles of the separatrices. In this case the manifold  $M^3$  admits a Heegaard decomposition of genus  $g_f$ .

We consider also some problems concerning topological classification of Morse-Smale diffeomorphisms and existing of Energy functions for them.

## **Asymptotic behavior of equilibrium measures corresponding to finite subgraphs of an infinite directed graph**

Boris Gurevich

Moscow State University and Institute for Information Transmission Problems, Moscow, Russia

The following question falls within the domains of Thermodynamic formalism for countable state symbolic Markov chains. Let  $X(G)$  be the doubly infinite path space of a connected digraph  $G$  and let  $f$  be a real function on  $X(G)$ . Assuming that  $G$  is infinite, consider an increasing sequence of finite subgraphs  $G_n$  of  $G$  that converges to  $G$ . If  $f$  is sufficiently 'smooth', then for each  $n$  there exists a unique equilibrium measure  $\mu_n$  corresponding to  $G_n$  and  $f_n$ , where  $f_n$  is the restriction of  $f$  to  $X(G_n)$ . What is the asymptotic behavior of  $\mu_n$ ? In

particular, does  $\mu_n$  converge to an equilibrium measure  $\mu$  corresponding to  $G$  and  $f$ , provided that such  $\mu$  exists? The answer to the last question is in general negative, the answer to the first one is in general unknown, but what is known today will be covered in my talk.

**Translation flows on  $\mathbb{Z}^d$  covers of compact translation surfaces**

Pascal Hubert

Université Paul Cézanne Aix-Marseille III, France

This talk will be an overview on the dynamics of linear flows on  $\mathbb{Z}^d$  covers of compact translation surfaces. In a different language, I will explain results on skew products over interval exchange transformations. This is a very active area. I will present the subject for non experts in the field. This talk will be a preparation for the talks by Troubetzkoy and Delecroix.

**Skew products with the interval fiber over Markov shifts (on a joint work with D. Volk).**

Victor Kleptsyn

CNRS, Lab. J.-V.Poncelet (Moscow, Russia), Université de Rennes-1 (France)

We consider skew products over a Markov shift with the interval fiber; our goal is to describe (in the best possible way) their dynamics. It turns out that for a generic skew product, its dynamics is rather similar to the dynamics of one interval diffeomorphism (it is known not to be the case for the skew products with the circle fiber), with some new effects occurring (like Kudryashov's bony attractors).

**Cesaro convergence of spherical averages for actions of Markov groups and semigroups**

Alexey Klimenko

Steklov Mathematical Institute, Moscow, Russia

Suppose that a Markov group acts on a probability space by measure-preserving transformations. We prove that then Cesaro means of spherical averages converge in  $L^p$  ( $p \in [1, \infty)$ ) and almost everywhere. It is a joint work with A. Bufetov and M. Khristoforov.

**Dynamics of  $\mathrm{SL}(2, \mathbb{R})$**

Erwan Lanneau

Centre de Physique Théorique, Marseille, France

This is a joint work with Duc Manh Nguyen (Bordeaux). Pseudo-Anosov homeomorphisms on translation surfaces with a prescribed involution give rise to "Prym eigenforms". The goal of this talk is to present some results about the  $\mathrm{SL}(2, \mathbb{R})$ -orbit closures of these forms.

**Geodesics in Teichmüller space equipped with Thurston's Lipschitz metric.**

Anna Lenzhen  
Université de Rennes-1, France

Thurston introduced in 1980's an asymmetric metric on Teichmüller space  $T(S)$  of a surface  $S$  by considering the best Lipschitz maps between hyperbolic structures. I will give a short survey of what is known and talk about some new results concerning the geodesics in  $T(S)$  equipped with this metric. This is joint work with Kasra Rafi and Jing Tao.

**Volume entropy for surface groups**

Jérôme Los  
CNRS, LATP, Université de Provence, Marseille, France

The volume entropy is defined for finitely presented groups and depends on the presentation. This number is easy to compute for free groups and for the free presentations. Otherwise no technics are available to compute or even evaluate it. In this work a method, based on combinatorial dynamics on the boundary of the group, is presented for the very simplest cases of hyperbolic groups, namely for "geometric presentations" of surface groups.

**Ergodic flows and some problems in analysis**

Alexander Prikhodko  
Moscow State University, Russia

This is a joint work with El H. El Abdalaoui.

A series of recent advances in the theory of ergodic flows and its applications to spectral theory is showing an interesting interplay between dynamical systems and problems of classical analysis. In this talk we focus on a construction of ergodic flows and Abelian group actions having the spectral type with the following property:  $\sigma * \sigma$  is absolutely continuous with respect to the Haar measure. This effect provides a new set of example of singular measures on the real line having an extremal power rate of the Fourier coefficients decay:  $c_n = O(n^{-1/2+\epsilon})$  first constructed in an explicit form by R.Salem in 1943.

**Stationary measures on homogeneous spaces**

Jean-François Quint  
CNRS, Université Paris-13, Villetaneuse, France

Let  $G$  be a Lie group,  $\Lambda$  be a lattice in  $G$  and  $\mu$  be a compactly supported Borel probability measure on  $G$ . Assume the image by the adjoint map of the subgroup  $\Gamma$  spanned by the support of  $\mu$  has semisimple Zariski closure with no compact factor. Then every  $\mu$ -stationary Borel probability measure on  $G/\Lambda$  is homogeneous, that is carried by an orbit of its stabilizer in  $G$ . This gives a classification of  $\Gamma$ -orbit closures in  $\tilde{G}/\Lambda$ . This is a joint work with Yves Benoist.

### **The exponential drift**

Jean-François Quint

CNRS, Université Paris-13, Villetaneuse, France

I will present one of the main tools used in the proof of the result stated above. This uses an equidistribution property related to Martingale convergence to get some new invariance of a probability measure.

### **Recent results on additive structure of multiplicative subgroups**

Ilya Shkredov

Moscow State University, Russia

In the talk we give a survey of old and new results on uniform distribution, additive properties and applications of multiplicative subgroups of finite fields.

### **Shadowing lemma for partially hyperbolic systems**

Sergey Tikhomirov

Freie Universität, Berlin, Germany

We say that a diffeomorphism  $f$  of a manifold  $M$  is partially hyperbolic, if the tangent bundle of  $M$  admits an invariant splitting  $E^s \oplus E^c \oplus E^u$ , such that  $E^s$  and  $E^u$  are uniformly hyperbolic and  $E^c$  is not. If  $E^c$  is empty, then the diffeomorphism is uniformly hyperbolic.

Shadowing lemma says that in hyperbolic systems any pseudotrajectory can be shadowed by an exact trajectory. We introduce notion of central pseudotrajectory and prove that in partially hyperbolic systems any pseudotrajectory can be shadowed by a central pseudotrajectory.

### **On the Ergodicity of Flat Surfaces of Finite Area**

Rodrigo Treviño

University of Maryland, College Park, USA

We prove an ergodic theorem for flat surfaces of finite area which have periodic or recurrent Teichmüller orbits. Periodic and recurrent, in this setting, means that the action of the diagonal subgroup of  $SL(2; \mathbb{R})$  on the complex structure can be "renormalized" in a sense through its Veech group. This result applies in particular to surfaces of infinite genus and finite area, and we apply our result to existing surfaces in the literature to prove that the translation flow on a surface corresponding to a periodic or recurrent Teichmüller orbit is ergodic.

### **Ergodicity of infinite translation surfaces**

Serge Troubetzkoy

Institut de Mathématiques de Luminy, Marseille, France

I will discuss joint work with David Ralston on the ergodicity in a.e. direction of infinite periodic square tiled translation surfaces. We show that the geodesic flow is generically ergodic when the infinite surface is a skew product over a finite square tile surface having one cylinder directions (for example the torus, any square tiled surface in  $H(2)$ , and others). We also give concrete examples of ergodic staircases.

### **Translation surfaces without convex presentations**

Barak Weiss

Ben Gurion University of the Negev, Be'er Sheva, Israel

A translation surface is (a cut and paste equivalence class of) a finite collection of polygons glued along parallel sides; if in this definition one may take one strictly convex polygon, then we say the surface has a convex presentation. Veech showed that the lattice surfaces obtained by gluing two regular  $n$ -gons do not have a convex presentation. The problem of finding all surfaces without convex presentations is closely related to the question of classifying  $SL(2, \mathbb{R})$ -orbit closures. In joint work with Samuel Lelievre, building on results of McMullen, we find the complete list in the stratum  $H(2)$ .

### **Diffeomorphisms obtained from endomorphisms**

Evgeny Zhuzhoma

Nizhny Novgorod State Pedagogical University, Russia

S.Smale introduced the solenoids in Hyperbolic Theory of Dynamical Systems. His construction was generalized by R.Williams. We suggest similar construction and consider some results.