

Fuss-Catalan and Raney distributions versus products of random matrices

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We explicitly find positive measures $P_s(x)$ whose n -th Hausdorff power moment is the sequence of Fuss-Catalan numbers, defined by $FC_s(n) = \frac{1}{sn+1} \binom{sn+n}{n}$, with $s = 1, 2, \dots$ and $n = 0, 1, \dots$. Two-parameter generalization of Fuss-Catalan numbers, the Raney numbers are defined by $R_{r,k}(n) = \frac{k}{rn+k} \binom{rn+k}{n}$, $r = 1, 2, \dots$, $k = 1, 2, \dots$ and $n = 0, 1, \dots$. We explicitly find positive measures $W_{r,k}(x)$ whose n -th Hausdorff moment is the sequence $R_{r,k}(n)$. We discuss analytically and graphically these measures and demonstrate that $P_s(x)$ generalize the Marchenko-Pastur distribution, and $W_{r,k}(x)$ is a natural extension of Wigner's semicircle law, both characterizing different forms of products of random matrices.