If instead we start with the assumption that lawless sequences are real objects of the intuitionistic continuum whose properties are determined by their relationship to the lawlike sequences as suggested by (ii), we are led to an extensional theory reminiscent of the theory of generic real numbers, which has a classical model. These "relatively lawless sequences" satisfy Kreisel's Axiom of Open Data (i) and a strong continuity principle (but not bar induction) and form a comeager subset of Baire space with classical measure zero.

Here we present an intuitionistic theory of lawlike and relatively lawless sequences as an extension of S. C. Kleene's axiomatization FIM of Brouwer's theory of the continuum, and extend Kleene's function-realizability interpretation of FIM to the new system. Like the classical model, this realizability interpretation depends on the (classically consistent) set theoretic assumption that a particular Δ^2_1 well-ordered subclass of Baire space is countable.

AN. A. MUCHNIK, *On the inclusion of the class* BPP *into the class* Π_2.
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The inclusion mentioned in the title was proved by P. Gács (see [1]). He constructed a formula $F(R, l)$ having the form

$$\forall x \in \{0,1\}^{p(l)}\, \exists y \in \{0,1\}^{q(l)}\, Z(R, l, x, y),$$

where R denotes the set of binary words having length l, p and q are polynomially bounded functions, and the predicate Z is decidable in linear time (in l, length(x), and length(y)); R is given to Z as an oracle. The formula F is constructed in such a way that if $|R| > \frac{2}{3}2^l$ then F is true, and if $|R| < \frac{1}{3}2^l$ then F is false.

We propose another formula F with the same property, which is simpler than in [1] and has smaller degrees of polynomial boundaries for p and q.

We consider words having length $l\alpha$ lb l, where α is a sufficiently large constant and lb denotes the binary logarithm. Let $U = \{0,1\}^{l\alpha \operatorname{lb} l}$. We define $S \subset U$ as a set of all words w such that if we consider w as a catenation of α lb l words having length l each, then the most part of these words belong to R. Probability considerations show that if $|R| > \frac{2}{3}2^l$ then $|S| > (1 - \beta^{-\alpha \operatorname{lb} l})|U|$, and if $|R| < \frac{1}{3}2^l$ then $|S| < \beta^{-\alpha \operatorname{lb} l}|U|$, where β is a constant. We choose α in such a way that $\beta^{-\alpha \operatorname{lb} l} < l^{-2}$.

We assume that a group structure is defined on U such that multiplication is easily computable. The formula $F(R, l)$ is defined in the following way: *for each sequence v which contains αl words that are elements of U there exists a word $y \in U$ such that for any member of v the product of this member and y belongs to $S(R)$.*

Let us prove that this F has the required property.

Assume that F is true. Then each sequence of αl words that are elements of U can be obtained from an appropriate sequence of αl words that are elements of $S(R)$, multiplying them by an element of U. Therefore, $|U|^{\alpha l} \le |S|^{\alpha l}|U|$, i.e. $(2^{l\alpha \operatorname{lb} l})^{\alpha l} \le |S|^{\alpha l} 2^{l\alpha \operatorname{lb} l}$, and hence $|S| \ge 2^{l\alpha \operatorname{lb} l - \operatorname{lb} l} = l^{-1}|U| > l^{-2}|U|$. So $|R| \ge \frac{1}{3}2^l$.

Assume that F is false. Then there exists a sequence of αl words that are elements of U, such that when multiplied by an arbitrary element of U it intersects with $U \backslash S(R)$. We choose an element in each intersection of this type. In this way we have chosen $|U|$ elements and each word is repeated no more than αl times. So the number of chosen words is bigger than $|U|/\alpha l > l^{-2}|U|$ (it is enough to consider $l > \alpha$). Therefore, $|S| < (1 - l^{-2})|U|$ and hence $|R| \le \frac{2}{3}2^l$.

REFERENCE

[1] M. Sipser, *A complexity-theoretic approach to randomness*, ***Fifteenth annual ACM symposium on theory of computing***, ACM, New York, 1983, pp. 330–335.

ROMAN MURAWSKI, *Application of satisfaction classes to the study of models of fragments of second order arithmetic.*
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Let PA denote Peano arithmetic and A_2^- second order arithmetic. Recall that a model M of PA is said to be *T-expandable* (where T is A_2^- or a fragment of it) iff there exists a family $\mathfrak{X} \subseteq \mathscr{P}(M)$ such that $(\mathfrak{X}, M, \in) \models T$.

We shall be interested in T-expandability for a particular fragment T_0 of A_2^-: it is A_2^- with comprehension restricted to Δ^1_1 formulas and Σ^1_1 scheme of choice.