The third international *Moscow–Vienna Workshop on Logic and Computation* takes place at V. A. Steklov Mathematical Institute of Russian Academy of Sciences in Moscow. It provides a podium for a presentation of the ongoing cooperation between Moscow and Vienna groups of researchers in the fields of mathematical logic and computation theory at Steklov Mathematical Institute, Vienna University of Technology and Moscow State University.

Program committee
S.I. Adian, M. Baaz, L.D. Beklemishev

Invited speakers
M. Baaz (Vienna), A. Beckmann (Vienna), L.D. Beklemishev (Moscow/Utrecht), C. Fermüller(Vienna), R. Iemhoff (Vienna), V.N. Krupski (Moscow), A. Leitsch (Vienna), G. Metcalfe (Vienna), An.A. Muchnik (Moscow), N. Preining (Vienna), C. Richter (Vienna), C. Schallhart (Vienna), A.L. Semenov (Moscow), V.B. Shehtman (Moscow), I. Shapirovskiy (Moscow), A. Talambutsa (Moscow), S. Terwijn (Vienna), J. van Oosten (Utrecht), H. Veith (Vienna), R.E. Yavorskiy (Moscow).

Conference location
Steklov Mathematical Institute, Gubkina 8, 119991 Moscow
Conference hall, 9-th floor, tel. 938-37-44 or 36-37.

Registration
Room 515, Monday, May 31, 9:30 am.

Sponsored by
Steklov Mathematical Institute (Moscow)
Kurt Gödel Society (Vienna)
ÖAD, Büro für Wissenschaftlich-Technische Zusammenarbeit

PROGRAM

Monday, 31.05 (Conference hall, 9-th floor)

Morning session. *Chair:* S.I. Adian

10:00–10:30

Matthias Baaz. *Propositional quantifiers on Kripke frames.* Joint work with **Norbert Preining**.

In this lecture we discuss the various concepts of intuitionistic quantified propositional logics w.r.t Kripke frames. We characterize the quantified propositional logic on Kripke frames of type w. Finally we describe the status of quantified propositional Gödel logic in this framework.

10:30–11:00

Lev D. Beklemishev. *On logical formalisms for quantum circuits: some open questions.* Joint work with **R. Brak**.

We consider the question of formulating a logic that relates to quantum circuits in roughly the same way that classical boolean logic relates to boolean circuits. We argue that the traditional quantum logic of Birkhoff and von Neumann does not provide an adequate answer. We propose a different logical formalism with two composition operations and formulate some open questions about it.

11:00–11:30

Valentin Shehtman. *New results on products of modal logics.*

A product is a natural combination of modal logics, which has been actively studied in recent decade. In this talk we present new results on the finite model property (fmp) for products and related results on decidable fragments of predicate logics. For the proofs we use a special version of the filtration method, "filtration via bisimulation". Some products have a stronger property – the "product fmp"; in this case the corresponding predicate logics have the fmp with respect to standard models.

11:30–11:55 Coffee-break (Room 530, 5-th floor).

Chair: M. Baaz

12:00–12:30

Chris Fermüller. *Revisiting Giles: On Physics, Dialogue Games, Lukasiewiz logic, and Hypersequents.*

Already in 1974 Robin Giles suggested a dialogue game theoretic interpretretation of (infinite valued) Lukasiewiz logic. Giles argued that his results show that Lukasiewiz logic is an adequate base for formal theories that take into acount 'dispersion' as it necessarily occurs according to quantum theory. We want to draw attention to this (widely neglected) work and show that there are strong connections to recent proof theoretic results on Lukasiewicz logic.

12:30–13:00

George Metcalfe. *Hypersequents and Fuzzy Logic.*

Fuzzy logics based on t-norms and their residua have been investigated extensively from a semantic perspective but a unifying proof theory for these logics has, until recently, been lacking. In this talk we show that a suitable proof-theoretic framework for fuzzy logics is provided by hypersequents, a natural generalization of Gentzen-style sequents. In particular we consider hypersequent calculi for the logic of left-continuous t-norms and related logics, and for logics based on the three fundamental continuous t-norms, Godel logic, Lukasiewicz logic, and Product logic.

13:00–13:30

Norbert Preining. *Countable orderings and continuous embeddability.* (Joint work with **Arnold Beckmann and Martin Goldstern**).

To settle the number of different countable Gödel logics, we analyze countable scattered linear orderings with respect to continuous embedabbility. We give an outline on how to use better quasi orderings to settle the number of different countable scattered linear orderings with respect to continuous embedabbility, which relates to the number of Gödel logics.

13:30–15:00 Lunch.

Afternoon session. *Chair:* L.D. Beklemishev

15:00–15:30

Arnold Beckmann. *Abstract Measures for low level complexity.*

What is the complexity of a computational problem? The concrete complexity of a computation can easily be defined by the amount of resources used in the computation like time and space. By abstract measures of complexity we understand ideal objects inherent in concrete complexity. In this talk we will discuss proof theoretic ordinals as a paradigm of abstract measures of complexity, how they fit to low level complexity (they do not) and what adequate adaptions are known of proof theoretic ordinals to the area of low level complexity (e.g. dynamic ordinals).

15:30–16:00

Sebastiaan Terwijn. *Logic and Medvedev degrees.*

In this talk we discuss some connections between structures arising in computability theory and constructive logic. The

Medvedev lattice can be seen as an implementation of Kolmogorovs idea of a "calculus of problems". The hope was to obtain a computational semantics for constructive (intuitionistic) logic. Although this goal was not immediately realized, a beautiful theorem of Skvortsova says that there are indeed factors of the Medvedev lattice that capture intuitionistic propositional logic IPC. We discuss some further algebraic properties of the Medvedev lattice bearing on this. We answer a question of Sorbi by providing a characterization of the join-irreducible elements of the Medvedev lattice and its nonuniform variant, the Muchnik lattice.

16:00–16:25 Coffee-break (Room 530, 5-th floor).

16:30–17:00

Jaap van Oosten. *Realizability and Filtered Colimits.*

The "effective topos" is the world of mathematics of Kleene's realizability. There is a functor from the world of classical sets into this topos. It will be explained why it is of importance to consider the question as to what filtered colimits this functor preserves. It turns out that the answer to this questien depends more on set-theoretic combinatorics than on recursive function theory.

17:00–17:30

Helmut Veith. *On Vacuous Satisfaction of Specifications.* (Joint work with **Marko Samer**).

In model checking, a specification is vacuously true, if some subformula can be modified without affecting the truth value of the specification. Intuitively, this means that the property expressed in this subformula is satisfied for a trivial reason, and likely not the intended one. In this talk, we argue that the common definition describes extreme cases of vacuity where the subformula indeed collapses to a constant

truth value. We suggest a refined notion of vacuity (weak vacuity) which is parametrized by a user-defined class of vacuity causes. Under this notion, a specification is vacuously true, if a subformula collapses to a vacuity cause. Our analysis exhibits a close relationship between vacuity detection and temporal logic query solving. We exploit this relationship to obtain vacuity detection algorithms in symbolic, automata-theoretic and multi-valued frameworks.

Tuesday, 1.06 (Conference hall, 9-th floor)

Morning session. *Chair:* C. Fermüller

10:00–10:30

Vladimir Krupski. *Reference constructions in the logic of proofs.*

It is known that the propositional logics of proofs are decidable whereas the predicate case is nonaxiomatizable at all – the natural definition of a predicate proof logic gives a theory which is not recursively enumerable. We propose an extension of the propositional proof logic language by the second order variables denoting the reference constructors (like "the formula which is proved by x"). The proof logic in this weak second order language is axiomatized via the calculus $\mathcal{FLP}_{ref}$, the *Logic of Proofs with References.* The decidability, arithmetical soundness and arithmetical completeness of $\mathcal{FLP}_{ref}$ are established. $\mathcal{FLP}_{ref}$ gives a flexible proof-theoretical interface for arithmetic. We show how the $\mathcal{FLP}_{ref}$-language can be used as a scheme language for both arithmetical lemmas and arithmetical inference rules specification. The $\mathcal{FLP}_{ref}$-provability gives the complete admissibility test for arithmetical inference rules provided the rules are in the scope of the specification method.

10:30–11:00

Rosalie Iemhoff. *Basic Intuitionistic Logic of Proofs.* (Joint work with **Sergei Artemov**).

The basic logic of proofs consists of the propositional schemes that are valid about the proof predicate $\mathit{Prf}(x, y)$. It is a logic in the language of propositional logic extended by expressions t:A, where t is an atom and A a formula, that are interpreted as "t is a proof of A". We will present an arithmetical completeness theorem for the basic intuitionistic logic of proofs, that is a completeness theorem w.r.t. Heyting Arithmetic. This result disproves the conjecture that the intuitionistic logic of proofs consists of intuitionistic logic extended by the characteristic axioms of the classical logic of proofs and the law of excluded middle for expressions t:A. We will briefly comment on the extension of the result to the case in which functions on proof terms are allowed.

11:00–11:30

Rostislav Yavorskiy. *On Kripke style models for provability logic with quantifiers on proofs.*

We consider extensions of the Logic of Proofs in the language with quantifiers over proof variables. One interesting open question in this field is decidability of the logic corresponding to the standard Goedel proof predicate.

As a natural step in this direction we introduce Kripke style semantics for this language.

Theorem. Let LPq denote the set of formulas in the language of provability logic with quantifiers on proofs, which are valid in all adequate Kripke models. The logic Lpq is decidable.

11:30–11:55 Coffee-break (Room 530, 5-th floor).

Chair: V. Shehtman

12:00–12:30

Alexei L. Semenov. *Methodologies of prediction.* (Joint work with **Andrei Muchnik**).

Abstract

12:30–13:00

Christian Schallhart. *Approximation Results for Sym-*

bolic Problem Descriptions. (Joint work with **Luca Trevisan**).

Symbolic representation of decision problems by Boolean formalisms is a powerful method for efficient algorithmic solutions in fields such as VLSI design. We present theoretical results which demonstrate that symbolic problems are hard to approximate. To this end, we employ a variant of the PCP theorem for the complexity class NEXP.

13:00–13:30

Ilya Shapirovsky. *PSPACE decidability of modal logics via selective filtration.*

We present a method allowing to construct a PSPACE decision procedure for some modal logics.

We show how this method works for two particular logics $\mathbf{L}_1$ and $\mathbf{L}_2$. These logics were introduced in the study of the chronological future relation in Minkowski spacetime [G,SS]. The finite model property for this logics was proved in [SS] using the Kripke–Gabbay method of selective filtration in a combination with the method of maximal points [F]. It allows us to reduce the satisfiability problem for these logics to the satisfiability problem on certain frames. In spite of the exponential size of these frames, the algorithm works within a polynomial space.

Also we show how to transfer our method to some other transitive logics.

Bibliography

[F] K. Fine. Logics containing K4. I. Journal of Symbolic Logic, v. 39(1974), 31-42.

[G] R. Goldblatt. Diodorean modality in Minkowski spacetime. Studia Logica, v. 39 (1980), 219-236.

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13:30–15:00 Lunch.

Afternoon session. *Chair:* A.L. Semenov.

15:00–15:30

Alexander Leitsch. *A resolution-based cut-elimination method (CERES) for Gentzen's LK.*

A resolution-based cut-elimination method (CERES) for Gentzen's LK is defined. First cut-elimination is generalized to the problem of redundancy-elimination. Then the elimination of redundancy in LK-proofs is performed by a resolution method in the following way: a set of clauses C is assigned to an LK-proof p and it is shown that C is always unsatisfiable. A resolution refutation of C then serves as a skeleton of an LK-proof p' with atomic cuts; p' can be constructed from the resolution proof and from p by a projection method. In the last step the atomic cuts are eliminated and a cut-free proof is obtained. The complexity of the method is analyzed and it is shown that a nonelementary speed-up over Gentzen's method can be achieved. The clause sets C can be represented by terms, so-called clause terms. This term representation is crucial in a mathematical comparison of CERES with a general class K of cut-elimination methods (containing that of Gentzen and Tait). It is shown that no method in K is capable of improving CERES nonelementarily. As the core of the method is a resolution theorem prover CERES turns out to be efficient also in practice.

15:30–16:00

Clemens Richter. *The Cut-Elimination Program CERES.*

CERES is a resolution-based method for the elimination of nonatomic cuts in LK-proofs. The use of resolution suggests a computational implementation able to handle rather complex cut-elimination problems. In this talk we present an implementation of CERES, in particular the basic algorithms, the input format, and the integration of the well-known automated theorem prover 'Otter'. Finally we demonstrate the program on some examples.

16:00–16:30

Alexey Talambutsa. *On growth function of languages with one excluded subword.*

Let $\mathcal{A} = \{a_1, a_2, \ldots, a_n\}$ be a finite alphabet. The set of all words in $\mathcal{A}^*$ that are not containing subword W is called *language over $\mathcal{A}$ with one excluded subword* W. The amount of words in this language having length equal to n is called *growth function*, denoted by $F_W(n)$.

We prove following results:

Theorem 1. Let V and W be words of the same length. If W is a power of a letter (i.e. $W = a_i^n$ for some letter a_i) then $F_W(n) \geq F_V(n)$ for every n.

Theorem 2. Let V and W be words of the same length. If V is a unbordered word (i.e. none of it's non-trivial prefix is a suffix), then $F_W(n) \geq F_V(n)$ for every n.

Corollary. If the word W is longer than word V then

$$F_W(n) \geq F_V(n) \text{ for every } n$$

17:00 **Reception.** (Room 530, 5-th floor.)