Lyapunov Spectrum with Arbitrary Many Inflection Points

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Lyapunov Spectrum

The *Lyapunov exponent* of a piece-wise differentiable mapping $T\colon [0,1] \to [0,1]$ is a function

$$\lambda(x) = \lim_{n\to\infty} \frac{1}{n} \log |(T^{\circ n})'(x)|$$

if the limit exists.

- The Lyapunov exponent is well defined on a set of full measure for any invariant measure μ;
- **L. Barreira & J. Schmelling, 2000:** For conformal repellers and SSFTs the set of points where the Lyapunov exponent does not exist has full Hausdorff dimension;
- For ergodic measure μ, we can define

$$\lambda_\mu = \int_0^1 \log |T'(x)| d\mu$$

- **H. Weiss, 1999:**

$$[0,1] = \{x \in [0,1] \mid \lambda(x) \text{ doesn't exist}\} \cup \cup \{x \in [0,1] \mid \lambda(x) = \lambda_\mu\} \cup \{x \in [0,1] \mid \lambda(x) \neq \lambda_\mu\}$$

The *Lyapunov spectrum* is the map given by

$$L(\alpha) = \dim_H \{x \in [0,1] \mid \lambda(x) = \alpha\}$$

Consider

- equilibrum measure μ_t
- entropy $h(\mu_t)$

Then

$$L(\alpha) = \frac{1}{\alpha} \inf_{t\in\mathbb{R}} \left(h(\mu_t) + t\alpha - t\int \log |T'| d\mu_t \right)$$

Using the pressure function

$$L(\alpha(t)) = \frac{h(\mu_t)}{\alpha(t)} = \frac{P(-t\log|T'|)}{\alpha(t)} + t$$

Lyapunov spectrum is an analytic function taking values in an open interval; in most numerical experiments it appears to have a unique local maxima.

Cookie Cutters

Given a set of pairwise disjoint closed intervals $I_k \in [0,1]$, we say that the map $T\colon \bigcup_{k=1}^N I_k \to [0,1]$ is *a cookie-cutter with N branches* if

- $T(I_k) = [0,1]$ for $k = 1, \dots, N$
- the map T is of the class $C^{1+\varepsilon}$ for an $\varepsilon > 0$
- $|T'| > 1$ for all $x \in \bigcup_{k=1}^N I_k \to [0,1]$.
- The cookie cutter is *linear* if $T\,|_{I_k}$ is affine for any $1 < k < N$.

Linear Cookie Cutters Formulae

Consider a linear cookie-cutter map T with n branches of slopes $a_1, \dots, a_n$. Then,

$$\alpha(t) = \frac{\sum_{i=1}^n |a_i|^t \log|a_i|}{\sum_{i=1}^n |a_i|^t}$$

$$L(\alpha(t)) = \frac{\log(\sum_{i=1}^n |a_i|^t)}{\alpha(t)} - t$$

Main Result (after O.J., M.P. & P.V.)

- For any $m \in \mathbb{N}$ there exists a linear cookie cutter with at least $2m$ inflection points.
- The construction is explicit and the resulting cookie cutter consists of approximately $\exp(5m^2)$ branches.
- The result is easily extends to the Gaussian cookie cutters consisting of a few branches of the continued fraction map $T_{k_1}, \dots, T_{k_N}$,

$$T_{k_j}(x) = \frac{1}{x} - k_j, \text{ for } x \in \left[\frac{1}{k_j+1}, \frac{1}{k_j}\right]$$

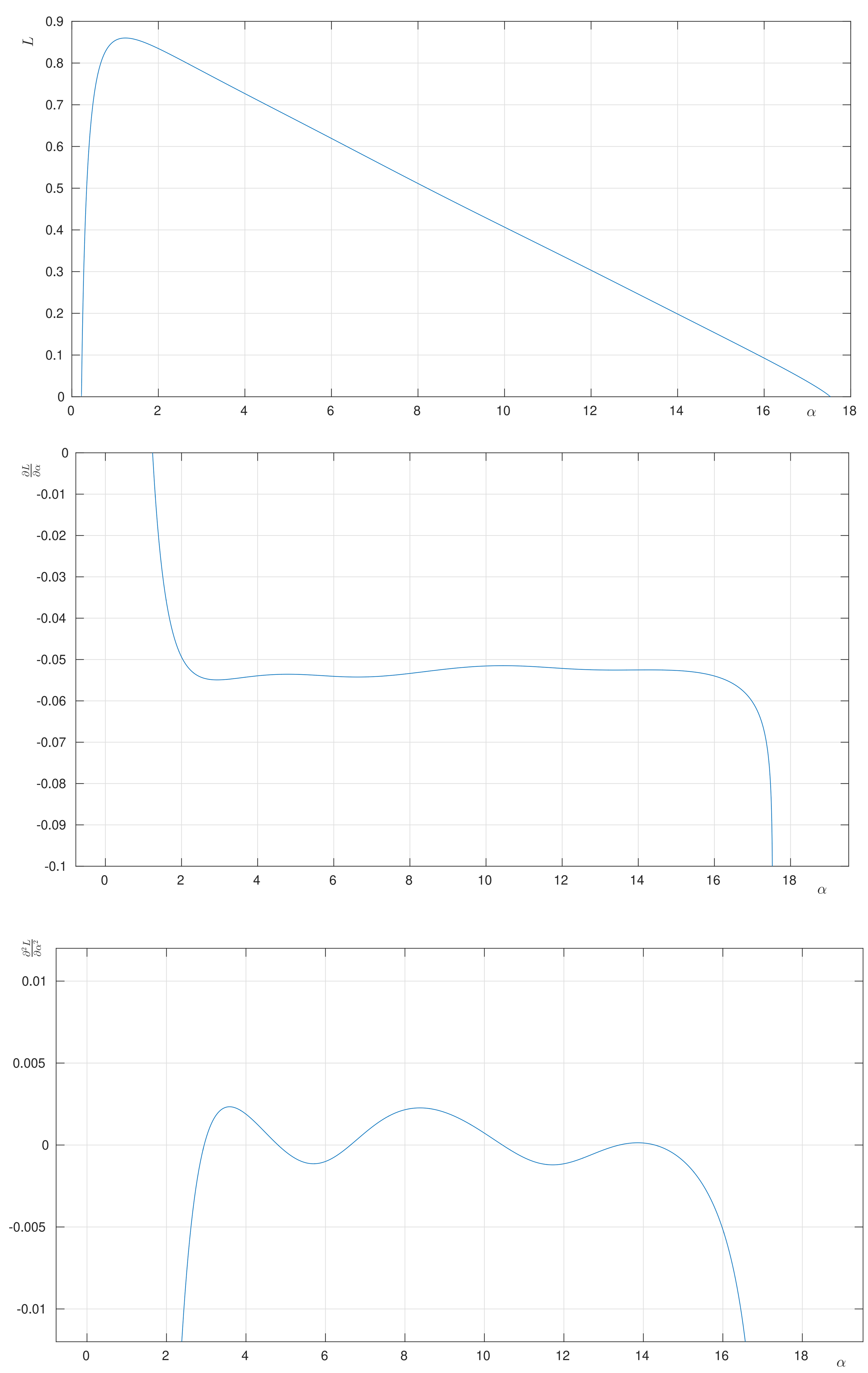

Figure 1: Linear cookie cutter with 62 branches and 6 inflection points