Paradox of choice in social network games with product choice

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(joint work with Nikita Nikitenkov, Moscow State University)

Paradox of choice:

Giving some players more options (or better payoffs) destroys the old equilibrium and the system switches to a new equilibrium which is worse for **all** players

For example, a set of individual improvements appears that leads into a worse equilibrium

It is easy to improve one player's situation and make everyone else worse off.

Braess's paradox

In a road network with travel time depending on congestion adding a new road can increase travel times for everyone.

Braess's paradox

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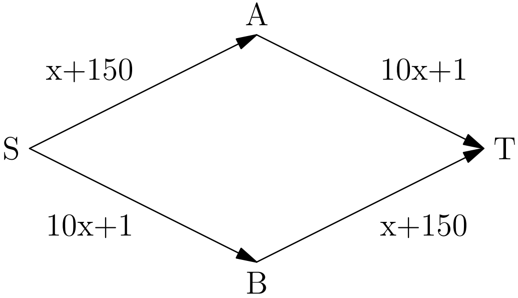
Consider the travel times:

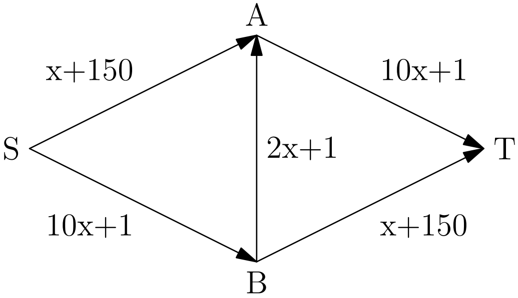


x is the number of cars (or thousands of cars) choosing this road

Even split, time is $6x + 151$.

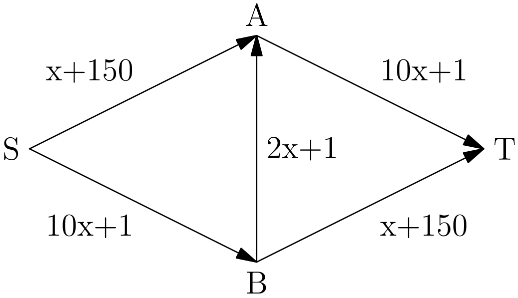
Braess's paradox

Adding a road:



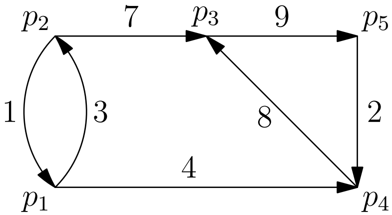
For $x \approx 10$ the switch to the empty shortcut with the cost $\approx 10x + 3 (\approx 103 < 211 \approx 6x + 151)$ becomes appealing

Braess's paradox



Everyone switches to the shortcut, time is now $22x+3 (\approx 223)$, but if one player switches back individually, time spent would be more than $10x+151 (\approx 251)$.

Social network games with product choice



Each vertex corresponds to a controlling player
Each player has a set of available products with different costs
Available strategies are: pick a product, pay the cost, and get positive bonus equal to the sum of incoming edges from the players with the same product; or refuse to choose a product and get zero payoff (no cost and no bonus).

Social network games with product choice

Introduced by K. Apt and S. Simon

Existence of a Nash equilibrium is NP-complete [S. Simon]

Related to the paradox of choice via individual improvements

Individual improvement properties first studied by K. Apt, S. Simon and E. Markakis

Individual improvement chains

Individual improvement for a strategy profile is a change of a single strategy that increases the payoff for the switching player

Nash equilibrium means that no individual improvements are possible

An individual improvement chain comes to a cycle or to an equilibrium (we assume that the player number is finite)

Social network games with product choice: price changes

Raising the price of a product to a value larger than the sum of all the edge weights in the graph is the same as forbidding the use of such a product

Similarly, allowing access to a product may be represented as lowering the price

Social network games with product choice: individual improvement chains

A **vulnerable** game: allowing one more strategy to one player (or lowering the price of one product for one player) creates individual improvement chains and each of them leads to a strictly worse (for all players) equilibrium

A **fragile** game: allowing one more strategy to one player creates individual improvement chains and each of them leads to a cycle

An **inefficient** game: raising a price of one product for one player creates individual improvement chains and each of them leads to a better equilibrium

An **unsafe** game: raising a price of one product for one player creates individual improvement chains and each of them leads to a cycle

Social network games with product choice: individual improvement chains

Apt, Simon, Markakis provided examples of fragile, unsafe and inefficient games
Also offered weaker notions of vulnerability (and provided examples)
One of the weaker definitions: obligatory product selection, there is no refusal to use any product

Existence of vulnerable games is proved in the presented work

Cascade

Cascade is a black-box construction for building examples of social network games with product choice

When the external conditions change, a cascade switches between two states by following the only possible individual improvement chain.

Cascade

There are three products; product costs are lower than any of the edge weights

Each player has at most two products available

Cascade

There is one incoming control edge, depending on the incentives provided by this edge cascade is in one of two possible equilibrium states

Changing the product choice at the origin of the incoming edge starts the **only** possible chain of individual improvements leading to the other equilibrium state

One of the states provides much higher payoffs due to special weak links («emotional» links); these payoffs can also be applied to external players without affecting their strategic choices

Cascade — the two states

The good state: the incoming control edge incentivizes the choice of the product A, the outgoing control edge also incentivizes the choice of product A

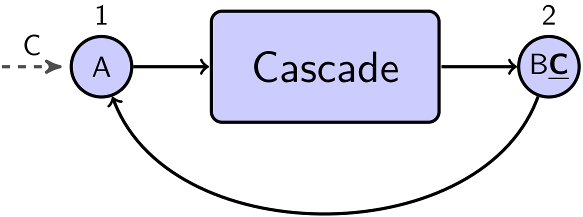
The bad state: the incoming control edge incentivizes anything else, the outgoing control edge incentivizes the choice of product B

Cascade — the interface details

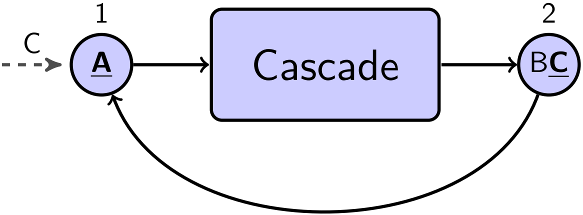
The product prices are low and equal.

The cascade also uses medium-weight «inclination» links to ensure that there is a product that is better than refusing to choose the product. In the next few slides these inclinations are also provided to the players outside the cascade; in this case they are shown by emphasizing one product in the list of options.

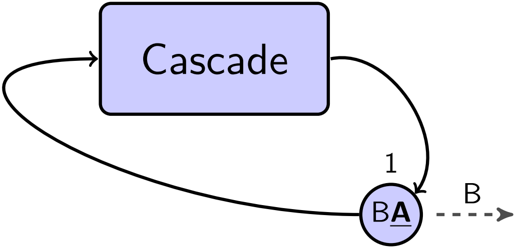
A vulnerable game



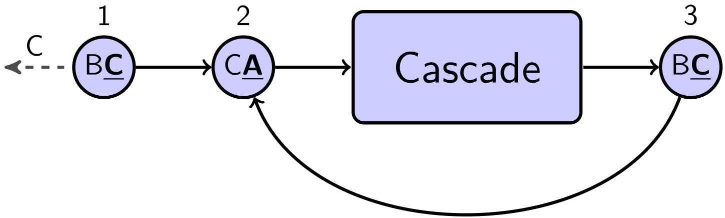
A fragile game



An inefficient game



An unsafe game

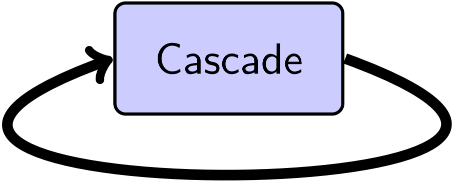


Edge changes

Instead of adding or removing products (or changing their prices) we can try to fine-tune the payoffs by changing the weight of a single edge (low enough edge weights are equivalent to edge removal)

Edge changes

The simplest example is universal reduction of payoff after an individual improvements chain started by reducing the weight of a single edge



Edge changes

You can either

- increase edge weight
  or
- decrease edge weight

and make the only individual improvement chain lead to:

- universally better equilibrium
- universally worse equilibrium
- individual improvement cycle

Cascade structure

Three products
Each player has at most two products available
Product costs are very low but positive
Three types of edges

Two types of players: «humans» and «spirits», spirits have no free will
Cascade is built out of three very similar ranks

Cascade — the construction idea

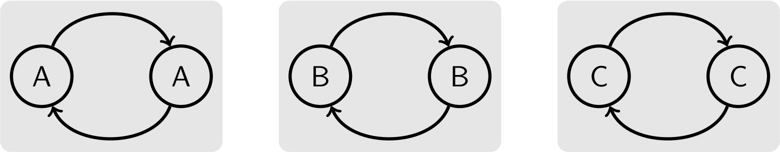
The main idea is having three types of edges:

- Emotional edges: very low weight, very numerous, never affect strategical choice
- Control edges: very high weight, often reward using products that are not available
- Inclination edges: high weight, but not as high as control edges, intended to reward using some product so that refusal strategy is not used

Cascade — switching the state

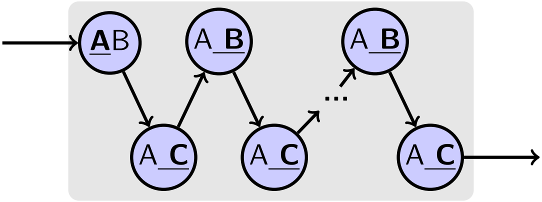
For every player incoming emotional links switch between equal incentives for both available products and incentive for an unavailable product; the difference between incentives for two available products is never higher than one emotional link weight

Cascade structure
Spirits



A-stimulus **B-stimulus** **C-stimulus**

Cascade structure
Ranks of humans



A-rank

Cascade structure
Cascade composition

Spirits provide inclinations

Main input controls a C-rank, which controls an A-rank, which controls a B-rank

Possible next step

Is there a less rigid/more believable construction?

Thanks for your attention!

Questions?